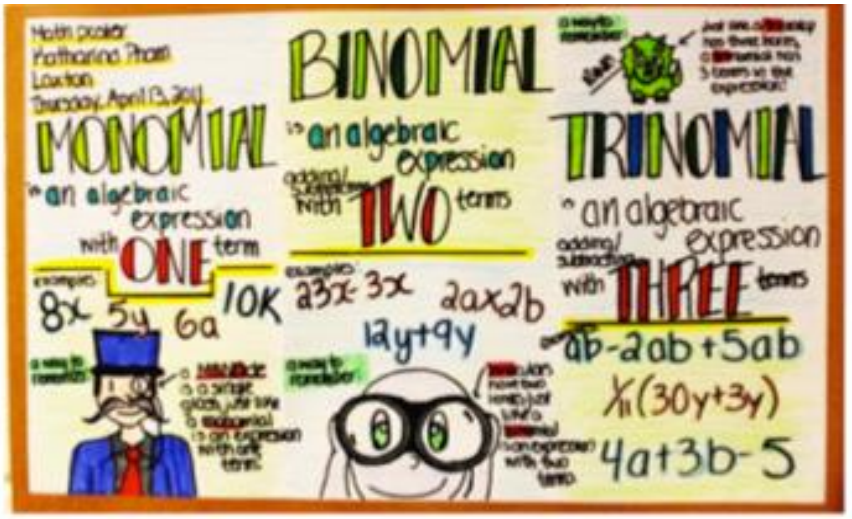


UNIT 5: PROBLEM

Creating A Comic Strip or Infographic Poster

What Drinking Box Design Is Most Efficient?

ALGEBRA | Grade 9 Academic Mathematics Student Workbook| MPM1D1

"Making decisions in light of gospel values with an informed moral conscience"

Name: _____

Teacher: _____

Cardinal Ambrozic C.S.S. Mathematics Department

Problem 5: What Drinking Box Design Is Most Efficient

MPM1D1
Grade 9 Academic Mathematics: Principles of Mathematics

Tool Number	Lesson Title & Topics	Topics	Homework
1	Exponents I	Introduction Lottery Problem Video of Universe Terminology Base/Exponent	3.2 #1 – 4, 5bdg, 6ad, 8, 11, 14
2	Exponents II	Multiply/Divide Power of a Power Fryer Model	3.3 #1,2bc, 3,4, #5, 6bd, 7bc, 8adei, 8acei, 10
3	Collecting Like Terms/Add and Subtract Polynomials	Intro Problem – Garden Box/Pool Table Degree Expression/Coefficient/Variable Polynomial Intro to Algebra Tiles SB Smart Response	3.4 #1 – 8, 9ac, 11, 12, 13 3.5 #3 – 5, 7ace, 8def, 12 3.6 #2ace, 4ace, 5fgh, 7, 8, 9
4	The Distributive Property	Intro Problem (Area) Intro Problem (23 x 7) TIPS Transfer Modelling Distribution (Table, Tiles, Dist) Nested Brackets	3.7 #3 – 5 ace, 7bf, 8ace, 9ace, 10, 13, 15fgh, 16cd, 17
5	Simplifying Polynomial Expression Review	Using different tools (Algebra Tiles, Virtual Tiles, CAS, paper and pencil) TIPS Transfer Course My Lesson CAS	
7	Summative Performance Task		
8	Pencil and Paper Task		

Parent/Guardian Signature: _____

Checklist
I understand and can correctly complete questions involving:

___ Ratios
___ Unit Rate
___ Solving Proportions
___ Percentage
___ Exponents
- Exponent Laws (multiply, divide, power of a power, negative exponent)

___ Polynomials
- Polynomial vocabulary (i.e. trinomial, variable, coefficient, constant…)
- Collecting like terms
- Adding/subtracting
- Distribution

MINDS ON

What considerations are made when designing a drinking box?

Extreme Collaboration

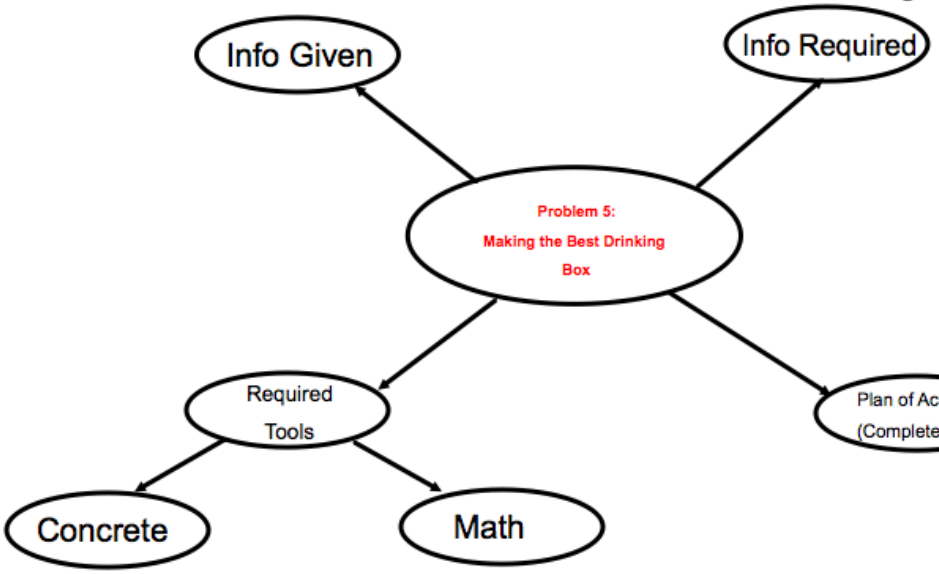
MINDS ON

Concept Map | Brainstorming

What dimensions of a drinking box use the least amount of materials to build?

Problem: *Algeropica* Juice Box is designing a new type of 250 cm^3 drinking box where the height is 5 cm more than double the width.

What dimensions of the box use the least amount of material to design?



Extreme Collaboration

Emphasize Relationships and why they are so useful in predications

Tool 5.1: Exponent Laws 1

MPM1D1: Principles of Mathematics

http://www.dpcdsb.org/AMBRO

Minds On: Think, Pair, Share

If someone offered you a choice of prizes which one would you accept:

1. One Million Dollars today **OR**
2. 1 cent today, 2 cents, tomorrow, 4 cents in two days, 8 cents… for 30 days (a month)

Think individually then discuss with your partner.

Minds On 2: The Universe (Video)

1. How many metres is 10^2 m? _____ Write 10^2 as a product of 10s._____
2. How many metres is 10^4 m? _____Write 10^4 as a product of 10s._____
3. How many metres is 10^8 m? _____Write 10^8 as a product of 10s._____
4. Explain how the exponent determines the number of meters.
5. How can you write 100000000000 as a power?
6. How many metres is 10^0 m? _____
7. How many metres is 10^{-2} m? _____
8. How many metres is 10^{-3} m? _____
9. How can you write 0.0000001 as a?

Action: Terminology

An expression of the form a^n is called a ***POWER*** .
A power has a **base** raised to an **exponent**.
A power is a short form for **repeated multiplication**.

a

Ex. 2^5 is read "two to the fifth". The power is _____
The base is _____
The exponent is _____

$2^5 = 2 \times 2 \times 2x 2 \times 2 =$
(The exponent 5 tells how many factors of 2 are multiplied together.)

Ex. 5^2 is read " _____"
4^3 is read " _____"

1. For each power, state the base, the exponent, and then evaluate:

a) 3^4 b) $(-6)^2$ c) -6^2 d) 10^1 e) $\left(\frac{3}{4}\right)^2$ f) $(-1)^2$ g) $(-1)^3$ h) $(-1)^{50}$

Note:	A negative base to an **EVEN** exponent = **POSITIVE** answer A negative base to an **ODD** exponent = **NEGATIVE** answer

2. Evaluate each power using the $\mathbf{y^x}$ key on your calculator.

a) 4^{10} b) $3(-2)^5$ c) $3^2 + 4^2$ d) $(3 + 4)^2$ e) $(-5)^2 - (-2)^4$ f) $2(-4)^2 \div (-2)^3$

3. Write each product in expanded form using powers:

a) 7 x 7 x 7 x 7 x 7 b) (-3) x (-3) x 9 x 9 x 9 c) $\left(\frac{5}{8}\right)\left(\frac{5}{8}\right)\left(\frac{5}{8}\right)\left(\frac{5}{8}\right)$

4. Write the first number as a power of the second:

a) 64, 2 b) 10000, 10 c) 81, -3 d) $\frac{1}{64}$, 4

Action: Laws 1, 2 and 3

Certain forms of bacteria double in population each day. If there are 1000 bacteria today, complete the chart below:

Time Passed (days)	**# Bacteria (thousands)**	**# Bacteria (in thousands) Write in <u>Expanded form</u>**	**# Bacteria (in thousands) <u>Power of 2</u>**
In 4 days: 4			
In 3 days: 3			
In 2 days: 2			
Tomorrow: 1			
Today 0	1		
Yesterday			
2 days ago:			
3 days ago:			
4 days ago:			

Exponent Laws 1, 2, and 3

1. a^n = _____

2. a^0 = _____ , $a \neq$ ___

3. a^{-n} = _____ , $a \neq$ ___

Ex. 1: Evaluate each power.

a) 5^0 b) $(-8)^0$ c) -8^0 d) 8^{-2} e) -5^{-3}

Ex. 2: Evaluate as a fraction in lowest terms.

a) $4^{-2} + 2^{-3}$ b) $\left(\frac{3}{5}\right)^0 - 5^{-2}$ c) $(7-3)^{-2}$ d) $\frac{2^{-1}+3^{-1}}{2^{-1}-3^{-1}}$

Ex. 3: Write as a power: a) $\frac{1}{27}$, base 3 b) $\frac{1}{32}$, base 2

HOMEWORK

3.2 #1 – 4, 5bdg, 6ad, 8, 11, 14

Product Rule (Multiplication of Powers)

1. Complete the following table.

Question	Expanded Form	Simplified Form	Evaluated
$2^2 \times 2^2$			
$5^2 \times 5^3$			
$5^2 \times 4^2$			
$3^4 \times 3^2$			
7×7^3			

2. Generalize your findings in your own words. Discuss with your group and develop a rule.

3. Generalize your findings using these algebraic expressions.

a) $x^2 \cdot x^3$ b) $x^2 \cdot y^2$ c) $a^m \cdot a^n$

d) $(4^{-3})(4^7)(4)$ e) $(2^2) + (2^3)$ e) $(4^2)(3^2)$

4. Create 2 questions of your own that verifies your understanding. Answer your questions.

1.
2.
3.
4.

Quotient Rule (Division of Powers)

1. Complete the following table.

Question	Expanded Form	Simplified Form	Evaluated
$\frac{2^3}{2^2}$			
$\frac{5^3}{5^2}$			
$\frac{5^2}{4^3}$			
$\frac{3^4}{3^2}$			
$\frac{7}{7^3}$			

2. Generalize your findings in your own words. Discuss with your group and develop a rule.

3. Generalize your findings using these algebraic expressions.

a) $5^7 \div 5$

b) $\frac{(-4)^5}{(-4)^3}$

c) $\frac{(-2)^2 x (-2)^4}{(-2)^3}$

d) $\frac{k^4}{k^7}$

e) $\frac{h^{-8}}{h^{-10}}$

f) $\left(\frac{3}{4}\right)^8 \div \left(\frac{3}{4}\right)^6$

4. Create 2 questions of your own that verifies your understanding. Answer your questions.

1.
2.

Power of a Power Rule

1. Complete the following table.

Question	Expanded Form	Simplified Form	Evaluated
$\left(2^2\right)^3$			
$\left(5^2\right)^3$			
$\left(3^4\right)^2$			
$\left(7^4\right)^3$			

2. Generalize your findings in your own words. Discuss with your group and develop a rule.

3. Generalize your findings using these algebraic expressions.

a) $(3^2)^6$ b) $(5^{-1})^2$ c) $\left[\left(-\frac{2}{3}\right)^2\right]^3$ d) $[(v^3)^6]^2$

Summary

Law #4: <u>The Product Rule</u>

To **MULTIPLY** powers with the **same** base, keep the base and _____ the exponents.

Law #5: <u>The Quotient Rule</u>

To **DIVIDE** powers with the **same** base, keep the base and _____ the exponents.

Law #6: <u>The Power of a Power Rule</u>

To find the power of a power, keep the base and _____ the exponents.

Examples

Simplify

$(3x)(2x)$ $(5x^2)(2x^4)$ $\dfrac{-2uv^3 \times 8u^2v^2}{(4uv^2)^2}$

$(4xy^3)(7x^3y^5)$ $(8x)(4y^7)$ $(3y^2)(y^6)(6x^3y)$

$(2x^2)(2x^6)(2x^8)$ $(3a^2b)^2 \div (ab)^2$ $(3x^3y^2)^3$

Fill in the boxes

$(\square x^2)\ (7x^{\square}) = 35x^6$

$(\square x^2y^{\square})(4x^{\square}y^4)(3x^8y^7) = 60x^{13}y^{12}$

$(abc)(2a^4b^5c^3)\ (\square a^{\square}bc^{\square}) = 2a^7b^7c^7$

The Pool Table Problem – Think, Pair, Share

A pool table is always twice as long as it is wide. Pool Cue Co. makes tables according to this relationship but in many different sizes. Each table top must have rubber bumpers around the outside edge and a felt top. The rubber bumpers cost \$2.25/m and the felt for the top costs \$28/m^2.

1. If the width is *x,* determine an expression to represent the total felt
2. If the width is *x,* determine an expression to represent the total rubber needed
3. Write a simplified expression to represent the total cost for the felt and rubber.
4. What is the cost of a table if it has a width of 1.5m

Polynomial Terminology

Terms

Identify the **variable, exponent, coefficient, constant term** and **term** below:

$$2x^2 + 5$$

Polynomials

Terminology	Examples
Polynomial — an algebraic expression consisting of one or more terms connected by addition or subtraction operators	
Binomial — An algebraic expression that contains TWO terms	
Monomial — An algebraic expression that contains ONE term	
Trinomial — An algebraic expression that contains THREE term	

Degree of a Polynomial

Degree of a term - the sum of the exponents on the variables in a term

Ex) Find the degree of the term below

a) $-4ab^3$ b) x^3yz^2

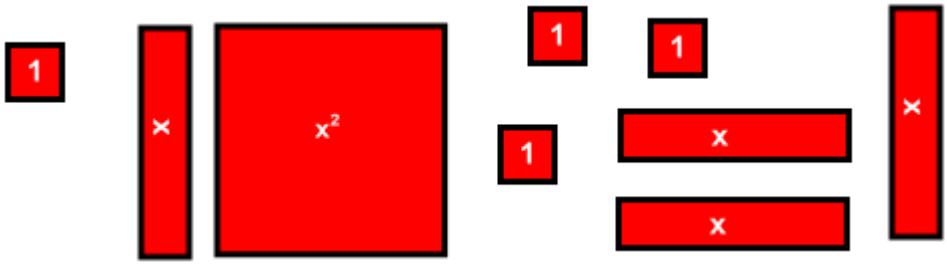
Degree of a polynomial - the degree of the highest term is the degree of the polynomial

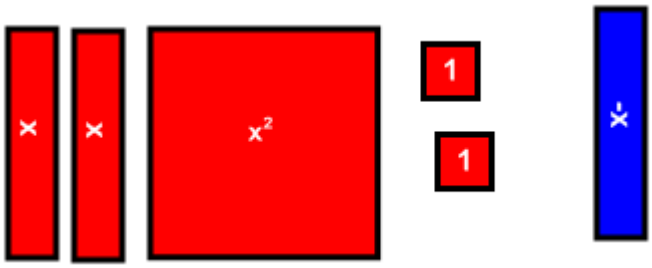
Polynomial	Degree of Polynomial
$x + 2$	
4	
$4x + 3x^2y - 3x^2$	
$5abc^3 + 3a^2b^2 - 4a^3bc^2$	

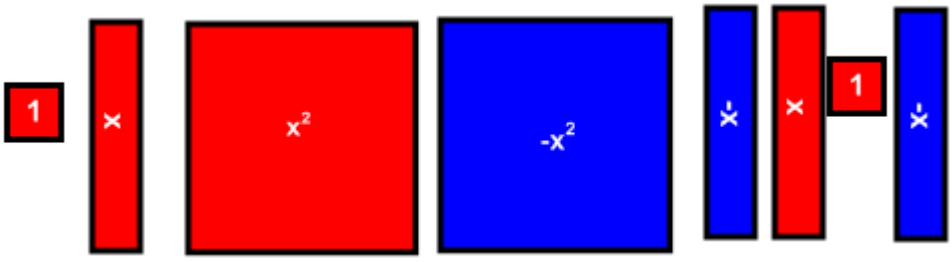
Simplifying Polynomials

Terms

Find the area of the tiles below







Like terms have the same variable and the same exponent

Example:

Adding/Subtracting Polynomials

Ex) Simplify the following polynomials

a) $2a + 3b – 5a + 6b$

b) $-3x^2 + 4xy – 4 – 2y^2 – 5x^2 + 10 – 9xy – 8y^2$

b) $(2a + 4) + (5a – 2)$

c) $(12a – 3) – (7a + 3)$

d) $(2x^2 + 3x) – (3x^2 – 4x + 4) – (x^2 – 5)$

Back To Problem

A pool table is always twice as long as it is wide. Pool Cue Co. makes tables according to this relationship but in many different sizes. Each table top must have rubber bumpers around the outside edge and a felt top. The rubber bumpers cost \$2.25/m and the felt for the top costs \$28/m^2.

1. If the width is *x*, determine an expression to represent the total felt
2. If the width is *x*, determine an expression to represent the total rubber needed
3. Write a simplified expression to represent the total cost for the felt and rubber.
4. What is the cost of a table if it has a width of 1.5m

Homework:
3.4 #1 – 8, 9ac, 11, 12, 13
3.5 #3 – 5, 7ace, 8def, 12
3.6 #2ace, 4ace, 5fgh, 7, 8, 9

What Does Graphing Tell Us About Algebra?

1. Complete the following tables of values:

a) $y = 3(x - 1)$

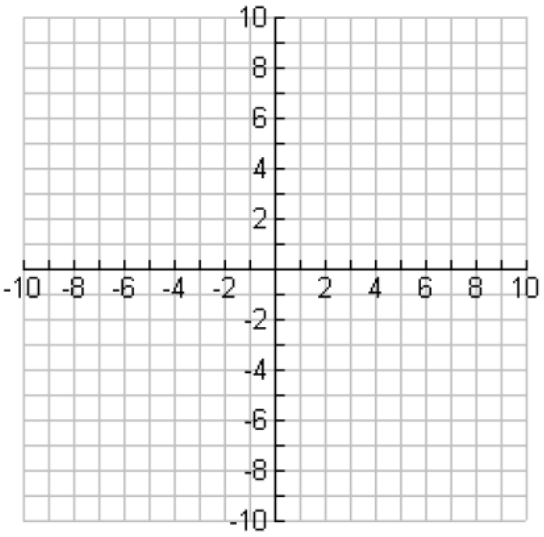
x	y
0	
1	
2	
3	
4	

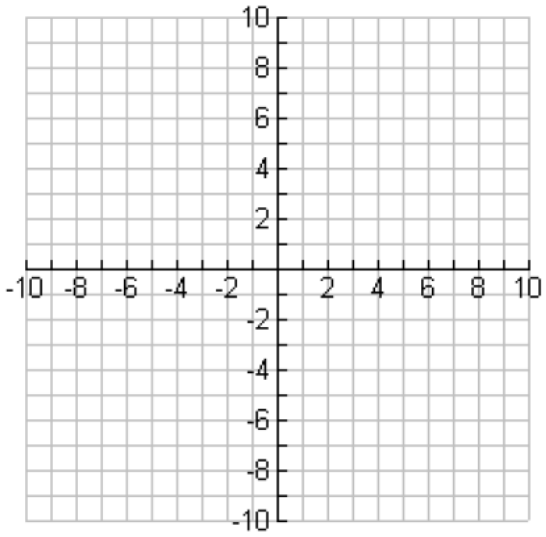
b) $y = 3x - 3$

x	y
0	
1	
2	
3	
4	

2. How do the tables compare? _____

3. Graph the two relations below.





4. What do you notice about the lines? Do you think the lines are the same or different? *(Make sure you make the scale the same)*

Distribution With Algebra Tiles – The Pool Table Problem

To maintain FIFA soccer field regulations a soccer field must have a length double the width plus 3 metres.

5. If the width was 2 m, determine the length.

6. Use tiles to represent the length and the width below

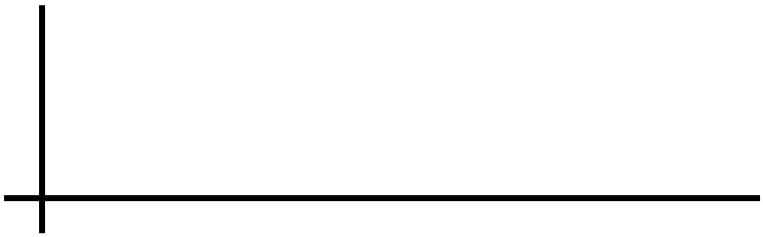
7. Fill the rectangle with tiles to determine the area of the field.

8. If the width is *x,* determine an expression to represent the width and the length

   a. Width = _____

   b. Length = _____

9. Use tiles to represent the length and the width below. Fill in the rectangle and determine the area of the field.

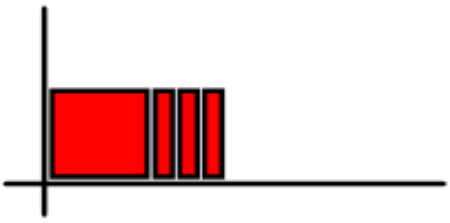
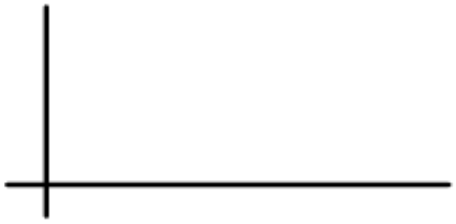


   Area = _____

10. If sod costs \$3/m^2, how much would it cost to sod a field with a width of 45 m.

11. Is there a more efficient way to determine the area of the soccer field?

The Distributive Property – Algebra Tiles

Rectangle	Width	Length	Area	A = l x w
	2	x+1		
				
				
	2x	x+3		

2. Look at the last column. Describe how you can start with the expression on the left side of the equation and get the expression on the right side.

3. Use the technique from step 2 to multiply the following.

a) $4(x + 3)$

b) $x(2x + 7)$

c) $3x(x + 2)$

The Distribution Property – Table Method

Expand and simplify the expressions.

Simplify **4(2x – 4)**

Simplify **-3(4x - 5)**

Simplify **7x(-9x + 5)**

The Distribution Property – Algebraic Method

Expand and simplify the expressions.

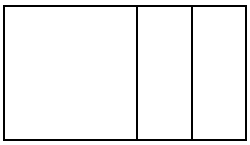
a) $-8(2 - d)$

b) $-5h(3h^2 - 7h - 2)$

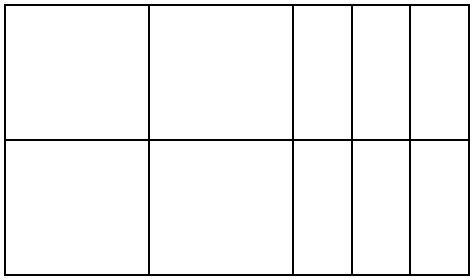
Examples

1. Use the area method to determine.

1. a) $y = x(x + 2)$

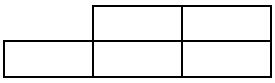


b) $y = 2x(2x + 3)$

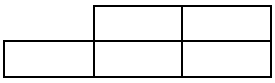


2. Use the chart method for multiplying a monomial by a binomial to expand and simplify the following:

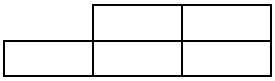
a) $x(x – 1)$



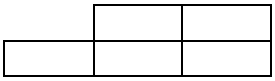
b) $x(x + 3)$



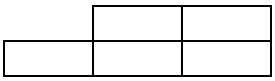
c) $2x^2(x^2 – 3x)$



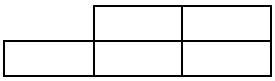
d) $2x^2(3x^2 + 2)$



e) $4x^2(x^3 – 3x + 2)$



f) $5x^3(2x^2 + 3x – 2)$



3. Use the distributive property to expand.

a) $x(x – 1)$ b) $2x(x + 5)$ c) $x^2(x – 1)$

d) $–3x(2x – 4)$ e) $2x^3(3x^2 + 2x – 4)$ f) $–5x^3(4x^3 – 7x + 3)$

3. Use the distributive property to expand.

a) $2x(x - 5) - 3x(x + 1)$

b) $\frac{2}{3}(3x-1)+\frac{1}{2}(4x+3)$

The Pool Table Problem – Think, Pair, Share

Simplify **2x(4x – 3)** using the method assigned by your teacher.

Method 1-Table Method

Method 2-Area Method-Algebra Tiles

Draw a sketch of your tiles below

Method 3-Algebraic Method

Expand the expression from above. Show all work

Joining Half One With Half Two

Using the simplified expressions from the class result simplify the following:

$2x(4x – 3) – 3x(x + 2)$

3.5.2: Half Class to Solve A Full Problem-Second Half (continued)

Simplify **-3x(x + 2)** using the method assigned by your teacher.

Method 1-Table Method

Method 2-Area Method-Algebra Tiles

Draw a sketch of your tiles below

Method 3-Algebraic Method

Expand the expression from above. Show all work

Joining Half One With Half Two

Using the simplified expressions from the class result simplify the following:

$2x(4x – 3) – 3x(x + 2)$

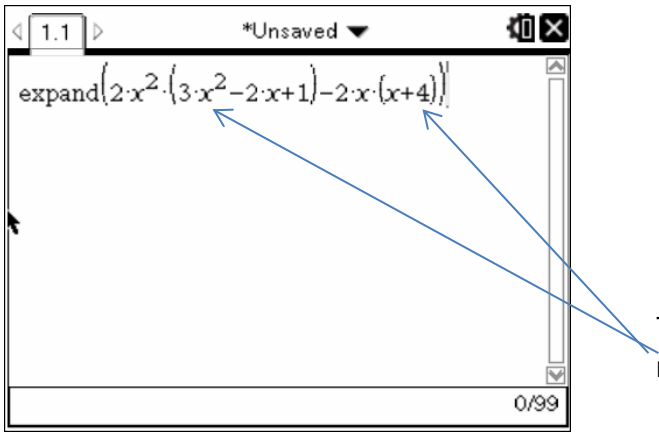
3.5.3: Checking Algebra Solutions

How To Check Your Answers With A CAS Calculator

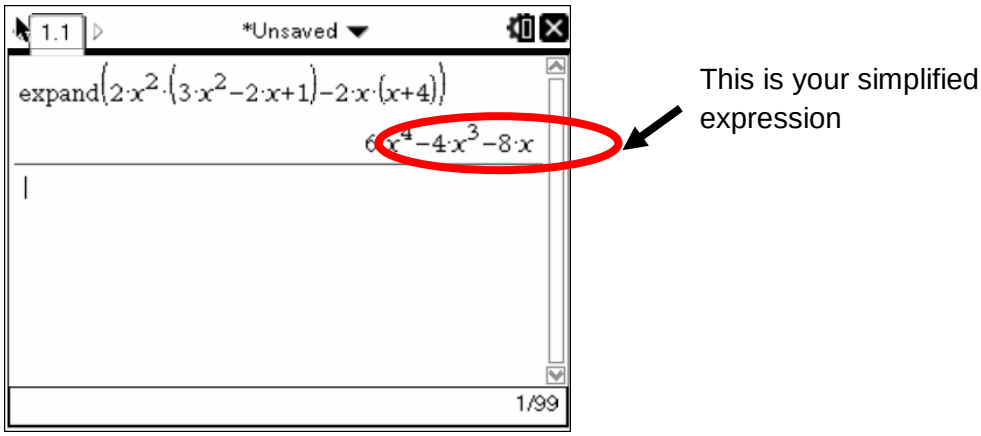
1. We will first simplify the expression: $2x^2(3x^2 - 2x + 1) - 2x(x + 4)$. Use the key "**^**" as your exponent button.
2. Turn on the calculator
3. Turn on the calculator and press the **HOME** button
4. Select **1:New Documents**
5. Select **NO** when it prompts you to save
6. Select 1:Add Calculator
7. Since distribution is the same as expanding type the following command with the key pad: **expand(**
8. It should like the screen below:

9. Now type in the expression you want to check:

$2x^2(3x^2 - 2x + 1) - 2x(x + 4)$



10. Press **ENTER** to expand.



3.5.4: Simplify-Join-Simplify-Check

You and your partner will simplify each side of each expression separately. You will then combine the expressions and simplify for a final answer. Finally you will check you solution using a CAS enabled calculator (use BLM 3.4.4 as a reference for your CAS calculator) **OR** you can check using Algebra Tiles.

Example 1) Simplify the following expression: $2x(3x – 4) + 3x(4x + 5)$

Partner A: $2x(3x – 4)$	**Partner B:** $+3x(4x + 5)$
Join and Simplify: $2x(3x – 4) + 3x(4x + 5)$ Check you answer using the CAS enabled calculator.	

Example 2) Simplify the following expression: $3x(x – 1) – x(4x + 6)$

Partner A: $3x(x – 1)$	**Partner B:** $– x(4x + 6)$
Join and Simplify: $3x(x – 1) – x(4x + 6)$ Check you answer using the CAS enabled calculator.	

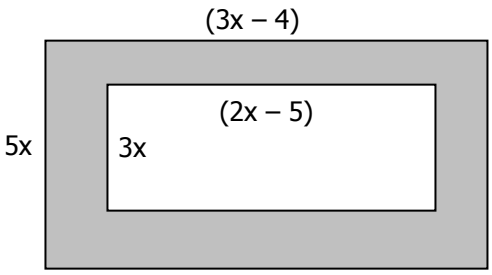
3.5.5: "Express"ions Yourself

Simplify the following expressions. Partner A will simplify the left column and partner B will check. The roles will be reversed for the right column.

PARTNER A	**PARTNER B**
$5(2x - 8) + 2(6x + 5)$	$7(2x - 8) + 4(2x - 9)$
$2x(3x - 3) + 2(x + 3)$	$3x(2x - 4) - 4x(5x - 2)$
$3x(2x - 2) - 2(x^2 + 4x - 4)$	$4x(3x - 5) + 3x(2x - 7)$
$5x(-4x + 5x - 7) - 4x(3x - 9 + 4)$	$x(7x + 6 - 10) - 3x(-8x - 9x + 4)$

3.5.6: More Algebraic Expressions Practice

One partner will answer each question and the other partner will check the solution. Alternate roles for each question.

Simplify the following expression: $2x(4x - 5) + 4x(3x - 7)$	Simplify the following expression: $4x^3(3x^3-2x^2+4x) - 2x(x^3-3x+4)$
Calculate the area of the shaded region:  	Design an expression and have your partner simplify it.
Is it possible to simplify all expressions? Explain. If you answered no, give an example of one that cannot be simplified.	What is wrong with the following solution? Explain $3x(3x-7)-4x(2x+6)$ $=9x^2-21x-8x^2+24x$ $=x^2+3x^2$ $=4x^2$

Problem 5

Problem: *Algeropicana* Juice Box is designing a new type of 250 cm^3 drinking box where the height is 5 cm more than double the width.

What dimensions of the box use the least amount of material to design?

Expectations

- Create an algebraic expression for the height, width and length of the box
- Create an equation to represent the volume of the box (expand and simplify)
- Create an equation to represent the surface area of the box (expand and simplify)
- Use the table below to determine the height and length of the box for the given widths below
- Use the table below to determine the surface are of the box at the given widths
- Highlight the dimensions on the table that gives the minimal surface area. Is this the best possible box dimension? Explain. You may use a graphical model in you explanation. (Surface Area is Dependent on Width)

Models

Volume	Width	Height	Length	Surface Area

1.RLS: Reflecting on Learning Skills

Students should be aware of the importance that these skills have on your performance. After receiving your marked assessment, answer the following questions. Be honest with yourself. Good Learning Skills will help you now, in other courses and in the future.

E – Always
G – Sometimes
S – Need Improvement
N – Never

Organization

E G S N I came prepared for class with all materials
E G S N My work is submitted on time
E G S N I keep my notebook organized.

Work Habits

E G S N I attempt all of my homework
E G S N I use my class time efficiently
E G S N I limit my talking to the math topic on hand
E G S N I am on time
E G S N If I am away, I ask someone what I missed,
E G S N I complete the work from the day that I missed.

Team Work

E G S N I am an active participant in pairs/group work
E G S N I co-operate with others within my group
E G S N I respect the opinions of others

Initiative

E G S N I participate in class discussion/lessons
E G S N When I have difficulty I seek extra help
E G S N After I resolve my difficulties, I reattempt the problem
E G S N I review the daily lesson/ideas/concepts

Works Independently

E G S N I attempt the work on my own
E G S N I try before seeking help
E G S N If I have difficulties I ask others but I stay on task
E G S N I am committed to tasks at hand

Yes No I know all the different ways available in my school, where I can seek extra help.

Yes No I tried my best.

What will I do differently in the next unit to improve?

_____
_____
_____
_____