

## Problem 2: How Do You Build A Kite

MPM1D1 - Geometry
Grade 9 Academic Mathematics: Principles of Mathematics

| Lesson Number | Lesson Title \& Topics | Topics | Homework |
| :---: | :---: | :---: | :---: |
| 2.1 | Geometry Introduction | - Triangle Theorem <br> - OAT <br> - C,F,Z-Pattern | Geometry Sheet |
| 2.2 | Angle Relationships in a Triangle Angle Relationships in Quadrilaterals | - Investigation on GSP | ```pg. 371 #1c, 2c, 4odd, 5odd, }1 pg. 381 #1bd, 5, 7, 10, 12``` |
| 2.3 | Angle Relationships in Polygons | - | $\begin{aligned} & \text { pg. } 391 \text { \#1b, 2b, 3b, 5, } \\ & 11,13 \end{aligned}$ |
| 2.4 | Midpoints and Medians in Triangles | - | $\begin{aligned} & \text { pg. } 398 \text { \#1, 2, } 7,8,9 \text {, } \\ & 11 \end{aligned}$ |
| 2.5 | Midpoints and Diagonals in Quadrilaterals | - | pg. 405 \#1, 3, 4, 6, 7 |
| 2.6 | Review-Formative Assessment | - Group Presentations of 5 problems <br> - Each group responsible for one problem | $\begin{aligned} & \text { Pg. } 408 \text { \#1,4,6ac, 7ac, 8, } \\ & 10,11 \end{aligned}$ |
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|  |  |  |  |
|  |  |  |  |

## Parent/Guardian Signature:

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## Checklist

I understand and can correctly complete questions involving:
interior angles of polygons
exterior angles of polygons
angle properties of parallel lines

## Quadrilaterals properties:

classify and draw sketches of Quadrilaterals.
identify Quadrilaterals by their diagonal properties.

## Project Kite Flyer

## Overview

In this project, your team:

- will build a kite that is at least 30 cm wide of ANY design (research online)
- will determine and explain all the geometric properties that are involved with the kite design.
- will involve the use of video to explain the geometric properties involved with the project.
- will design a kite with deep geometric analysis
- will design a kite that works


## Expectations

1. Sketch Design of Kite with Geometric Properties
2. Working Kite
3. Video Script
4. Video

## Geometric Tools

Parallel Lines - Alternate Angles (Z-Pattern)
Parallel Lines - Co-Interior Angles (C-Pattern)
Parallel Lines - Corresponding Angles (F-Pattern)
Supplementary Angle Theorem
Complimentary Angle Theorem
Opposite Angle Theorem
Isosceles Triangle Theorem
Equilateral Triangle Theorem
Interior Angles of Triangles
Interior Angles of Polygons
Exterior Angles of Polygons
Midsegments of Triangles
Medians of Triangles
Properties of Quadrilaterals


Member:

Oversees the project and keeps team on task and organized. Only person who can communicate with teacher.

- Make sure the following is handed in to teacher:
- Sketch of Kite
- Sketch of Geometric Properties
- Script for Video
- Make a neat copy of the team's Kite Journal.
- Use appropriate labels as necessary.
- Oversees construction.
- Complete silhouette information and display properly in room.
- Assist other team members as needed.


## Project Kite Flyers CHIEF ENGINEERS

Company Name:


## Member: <br> Member:

The leader in designing and assembling the kite

- Can have two separate duties: Engineer-designs model; Designer-Constructs Kite
- Creates sketch of design
- In charge of determining all the hardware and equipment needed
- Makes any adjustments during the building of the kite
- Adjusts and fixes the kite during flight date
- Kite innovations and new kite design is determined by the Chief Engineer and Designer
- Assist other team members as needed.


## Project Kite Flyer VIDEOGRAPHERS

## Company Name:



## Member: <br> Member:

The members that are in charge of creating the video describing the geometry in the kite design. This job can be broken down to two positions: Videographer and Director

- Research geometric terminology that applies to the kite design
- Use a video camera or phone camera
- Use software (iMovies or Voice Thread) to edit video and audio
- Post video to make it accessible


## Project Kite Flyer

 NARRATORCompany Name:


Member:

The narrator works with the videographer and creates a script of the geometry that is found in the kite design

- Creates script for video
- Works together with videographer
- Communicates with engineer to determine all the geometric properties involved in kite design


## Tool 2.1: Plane Geometry | Theorems

MPM1D1: Principles of Mathematics
http://www.dpcdsb.org/AMBRO

| 1) Supplementary Angles |  |
| :---: | :---: |
| 2) Complementary Angles <br> Given: $\quad \angle A O C$ is a right angle. <br> Explore: Measure $\angle A O B$ and $\angle C O B$. <br> How dol...? <br> Calculate the sum of these two angles. <br> Animate the diagram by pressing the button below. You can pause the animation by pressing the button again. <br> Conclude: What is the sum of the two angles? |  |
| 3) Opposite Angle Theorem |  |


| 4) The Sum of the Interior Angles of a Triangle |  |
| :---: | :---: |
| 5) Isosceles Triangle Theorem <br> Given: Isosceles $\triangle A B C$, where $A B=A C$ <br> Explore: Measure the angles opposite the equal sides ( $\angle A B C$ and $\angle A C B$ ) <br> How do l...? <br> Drag each of the vertices. <br> Conclude: What do you notice about the angles opposite the equal sides? |  |
| 6) Equilateral Triangles <br> Given: $\quad \triangle A B C$, where $A B=A C=B C$ <br> Explore: Measure each of the interior angles then drag point C . <br> How dol...? <br> Conclude: What do you notice about each of the interior angles? |  |
| 7) Exterior Angle Theorem <br> Given: $\triangle A B C$ and exterior angle $\angle A B D$. <br> Explore: Measure the non-adjacent angles $\angle A C B \& \angle C A B$. [How do 1.2 Calculate the sum of these two angles. Measure the exterior angle $\angle \mathrm{ABC}$. Drag point $A$. <br> Conclude: What do you notice about the sum of the two non-adjacent angles, $\angle A C B$ \& $\angle C A B$, inside the triangle and the exterior angle, $\angle A B C$ ? |  |


| 8) Parallel Line Theorem - Corresponding Angles <br> Given: $\quad \mathrm{AB}$ is paralle to $C D$ with transversal $P Q$. <br> Explore: Measure the corresponding angles $\angle B W X$ and $\angle D X Q$. <br> Howdol...? <br> Drag points B and X . <br> Conclude: What do you notice about the corresponding angles $\angle B W X$ and $\angle D X Q$ ? <br> More... Explain why this relationship is called the F-pattern. <br> There are four "F-patterns" in this diagram. Can you find them? $\square$ |  |
| :---: | :---: |
| 9) Parallel Line Theorem - Alternate Angles <br> Given: $\quad A B$ is parallel to $C D$ with transversal $P Q$. <br> Explore: Measure the alternate angles $\angle \mathrm{BWX}$ and $\angle W X C$. How do 1...? <br> Drag points B and X . <br> Conclude: What do you notice about the alternate angles $\angle \mathrm{BWX}$ and $\angle \mathrm{W} X C$ ? <br> More... Explain why this relationship is called the Z-pattern. <br> There are two "Z-patterns" in this diagram. Can you find them? Help |  |
| 10) Parallel Line Theorem - Co-interior Angles <br> Given: $\quad A B$ is parallel to $C D$ with transversal $P Q$. <br> Explore: Measure the co-interior angles $\angle B W X$ and $\angle W X D$. <br> How do $1 .$. ? <br> Calculate the sum of these two angles. Drag points B and X . <br> Conclude: What do you notice about the sum of the co-interior angles $\angle \mathrm{BW} \times$ and $\angle \mathrm{W} \times D$ ? <br> More... Explain why this relationship is called the C-pattern. <br> There are two "C-patterns" in this diagram. Can you find them? <br> Help |  |

### 2.1.P: Practice Sheet (continued)

Define each principle and determine the unknown angles.
1.

2.


$$
r^{\circ}=\quad \text { Reason: }
$$

3. 


4.


$$
\begin{array}{rlrl}
q^{\circ} & = & & \text { Reason: } \\
& = &
\end{array}
$$

5. 


6.


$$
\begin{aligned}
a^{\circ} & = \\
& =
\end{aligned}
$$

7. 



$$
\begin{aligned}
x^{\circ} & = \\
& =
\end{aligned}
$$

8. 



$$
\begin{aligned}
x^{\circ} & = \\
& =
\end{aligned}
$$

$$
m^{\circ}=
$$

Reason:


$$
w^{\circ}=
$$

9. Find the measure of a. Give reasons.

10. Find $x$. Give reasons.

11. Find the value of $x$.

Give reasons.

7. Find the values of the missing angles. Give reasons.

11. The diagram shows two parallel lines cut by a transversal. The measure of $\mathbf{a}+\mathbf{b}$ is
$\qquad$ . Give reasons.

12. For the following diagram, list as many examples of each Angle Theorem as possible.

13. Solve for $x$ and $y$


## Tool 2.2: Angle Relationships in Triangles and Quadrilaterals MPM1D1: Principles of Mathematics http://www.dpcdsb.org/AMBRO

## MINDS ON

## Tool 2.2 Team Problem

Eric's grandmother is sewing a quilt for him. A quilt consists of a piece of fabric of different shapes sewn together. The quilt that she is making will be use rhombus pieces of fabric joined together to make a star pattern.

An example of an eight-pointed star is shown. These rhombus pieces have an angle of $45^{\circ}$ at the centre.

Eric's grandmother makes star patterns with rhombuses that have an angle of $40^{\circ}$ at the centre. How many points will her star have? Justify your answer

Create your own quilt star-pattern using a centre angle that you think may work.
Explain why your pattern works.

1. Information Given
2. Information Needed
3. Tools Needed
a) Concrete Tools

b) Math Tools
4. Action Plan

## We Will Come Back To Problem

## Angle Relationships in Triangles



The sum of the interior angles of a triangle:


The sum of the exterior angles of a triangle:

## Angle Relationships in Quadrilaterals



The sum of the interior angles of a quadrilateral:

The sum of the exterior angles of a quadrilateral:

## Examining a parallelogram:



1. Determine the measure of each indicated angle and state reasons.
a)



2. Determine the values of $x, y$, and $z$. Give reasons.
a)

b)

c)

3. Determine the measures of $a$ and $b$. Give reasons.
a)

b)


Homework:
pg. 371 \#1c, 2c, 4odd, 5odd, 10
pg. 381 \#1bd, 5, 7, 10, 12

Investigate using Geometer's Sketchpad to complete the following table:

| Polygon | Number of Sides | Sum of Interior Angles | Sum of Exterior Angles |
| :--- | :--- | :--- | :--- |
| Triangle |  |  |  |
| Quadrilateral |  |  |  |
| Pentagon |  |  |  |
|  |  |  |  |
|  |  |  |  |

What pattern do you notice when calculating the sum of the interior angles?

What pattern do you notice when calculating the sum of the exterior angles?

For a polygon with $n$ sides, the sum of the interior angles is:

The sum of the exterior angles of a convex polygon is:

Example 1: Harmeet is building a deck in the shape of a regular octagon (all angles are equal). Determine the measure of the interior angles of the deck.


Example 2: A Canadian $\$ 1$ coin (also known as a loonie) is a regular polygon with 11 sides, called an undecagon.
a) What is the sum of the interior angles of the loonie?
b) What is the size of one of the interior angles?


Example 3: The sum of the angles of a regular polygon is $2340^{\circ}$. How many sides does it have?

## MINDS ON

## Team Problem-Think, Pair, Share

 How can you fold a triangle to rectangle with only three folds?

Explain the process!!
Give each pair a cut out of a triangle


## Investigate

Investigate and check using Geometer's Sketchpad:


1. Sketch the midpoint of $A B$ and label it " $D$ ".
2. Sketch the midpoint of $A C$ and label it "E".
3. Draw line segment DE.

## Hypothesize:

a) How do you think the lengths of DE and BC are related?
b) How do you think $\angle \mathrm{EDB}$ and $\angle \mathrm{DBC}$ are related?
4. Draw a perpendicular line from vertex $A$ to where it meets side BC. (This is the height of both of the triangles).

## Hypothesize:

c) How are the height of $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ADE}$ related?

## Investigate:



1. Find the midpoint of line segment BC and label it " D ".
2. Draw the line segment connecting vertex " $A$ " and point " $D$ ".

## Hypothesize:

a) What do you notice about BD and DC?
b) What do you notice about the area of $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ADC}$ ?

## Summary

## Examples

Edward's construction company is building a bridge across the lake from Point A to Point B. What length must they build the bridge according to the diagram?


Elise is sewing a strip of ribbon on a flag shaped like an isosceles triangle. The base measures 42 cm and the slant sides measure 72 cm . If she wants to sew the ribbon halfway between the base and the point, what length of ribbon should she cut for the flag?


Xenia is designing a large sail, in the shape of an isosceles triangle, for a model sailboat. The length of the median to the base side is 63 mm . The midsegment parallel to the base is 16 mm long. What is the length of the slant sides of the sail?


Homework:
pg. 398 \#1, 2, 7, 8, 9, 11
2-23|Page

## Tool 2.5: Midpoints and Diagonals in Quadrilaterals

MPM1D1: Principles of Mathematics
Textbook Chapter 7 Section 7.5
http://www.dpcdsb.org/AMBRO
MINDS ON: Ancient Roman Architecture


1. What is a lintel?
2. What is the shape of a lintel?
3. How does the arch overcome all of the problems of a lintel?
4. What shapes make the arch?
5. What massive Roman construction dependent on the arch design?
6. Explain how quadrilaterals were so important in Roman design.

## Investigating Quadrilaterals

1. Turn on automatic labelling of points. From the Edit menu, choose Preferences. Click on the Text tab, check For All New Points, and click on OK.
2. Construct any quadrilateral $A B C D$ by selecting the line segment tool $\qquad$

3. Construct the midpoints of the four sides by selecting the sides and then choosing Midpoints from the Construct menu.
4. Construct line segments EF, FG, GH, and HE by choosing Segments from the Construct menu. What type of quadrilateral does EFGH appear to be?
5. Measure and compare the sides of the smaller quadrilateral by choosing Length from the Measure menu.
6. What relationships are there among these lengths?
7. Deselect any object by clicking on any white part of the screen.
8. Measure the interior angles of quadrilateral EFGH. For example, to measure angle EFG, select vertices $\mathbf{E}, \mathbf{F}$, and $\mathbf{G}$ in that order. Repeat for angle FGH, GHE, and HEF?
9. Based on the angles and side lengths, what type of quadrilateral is this? Explain.

## Investigating Quadrilaterals Part 2

1. Download the file 2.5 Investigating Quadrilaterals.gsp from My Resources $\rightarrow$ Grade 9 Academic Mathematics $\rightarrow$ Unit 2: Geometry section of the course website.
2. Using the tabs on the bottom of the screen, investigate the 6 quadrilaterals by measuring the side lengths and anlges of the interior (smaller) quadrilateral. The interior quadrilateral was created by connecting the midpoints of the original shape.
3. Complete the table below:
$\left.\begin{array}{|l|l|l|l|}\hline \text { Quadrilateral Type } & \begin{array}{l}\text { Side } \\ \text { Measurements }\end{array} & \begin{array}{l}\text { Angle } \\ \text { Measurements }\end{array} & \begin{array}{l}\text { Identify The } \\ \text { Interior } \\ \text { Quadrilateral }\end{array} \\ \hline \text { Quadrilateral } & \begin{array}{l}\text { Opposite side } \\ \text { lengths are equal }\end{array} & \begin{array}{l}\text { Opposite angles } \\ \text { are equal }\end{array} & \text { Parallelogram }\end{array}\right]$


## Investigating Diagonals of Parallelogram

4. Select the Diagonals of a Parallelogram tab.
5. Line segments $\mathbf{A C}$ and $\mathbf{B D}$ are the diagonals of parallelogram $\mathbf{A B C D}$.
6. Measure the lengths of line segments $\mathbf{A E}, \mathbf{B E}, \mathbf{C E}$ and $\mathbf{D E}$. What do you notice?

Note: If two line segments divide each other into equal parts, they bisect each other.

## Examples

Calculate the lengths of $\mathrm{BE}, \mathrm{CE}, \mathrm{AC}$, and BD .


Homework:
pg. 405 \#1, 3, 4, 6, 7
Pg. 408 \#1,4,6ac, 7ac, 8, 10,11

## Exit Ticket

A flag is made from a rectangle with an inscribed quadrilateral created from the mid segments of each side. The size of the rectangle is 80 cm by 100 cm .

How much will the flag cost if
-the material for the inscribed quadrilateral costs $\$ 0.50 / \mathrm{cm}^{2}$
-the material that make up the triangle costs $\$ 0.75 / \mathrm{cm}^{2}$
-The border creating the inscribed quadrilateral costs $\$ 0.25 / \mathrm{cm}$


