Polya's Problem Solving Techniques

In 1945 George Polya published the book *How To Solve It* which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this book he identifies four basic principles of problem solving.

Polya's First Principle: Understand the problem

This seems so obvious that it is often not even mentioned, yet studens are often stymied in their efforts to solve problems simply because they don't understand it fully, or even in part. Polya taught teachers to ask students questions such as:

- Do you understand all the words used in stating the problem?

- What are you asked to find or show?

- Can you restate the problem in your own words?

- Can you think of a picture or diagram that might help you understand the problem?

- Is there enough information to enable you to find a solution?

Polya's Second Principle: Devise a plan

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation

- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious

Polya's Third Principle: Carry out the plan

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work discard it and choose another. Don't be misled, this is how mathematics is done, even by professionals.

Polya's Fourth Principle: Look back

Polya mentions that much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems.

So starting on the next page, here is a summary, in the master's own words, on strategies for attacking problems in mathematics class. This is taken from the book, *How To Solve It*, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-691-08097-6.

1. UNDERSTAND THE PROBLEM

- **First.** You have to *understand* the problem.
- What is the unknown? What are the data? What is the condition?
- Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

2. DEVISING A PLAN

- **Second.** Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.
- Have you seen it before? Or have you seen the same problem in a slightly different form?
- *Do you know a related problem?* Do you know a theorem that could be useful?
- *Look at the unknown!* Try to think of a familiar problem having the same or a similar unknown.
- *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem, try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

3. CARRYING OUT THE PLAN

- **Third.** *Carry out* your plan.
- Carrying out your plan of the solution, *check each step.* Can you see clearly that the step is correct? Can you prove that it is correct?

4. LOOKING BACK

- **Fourth.** *Examine* the solution obtained.
- Can you *check the result*? Can you check the argument?
- Can you derive the solution differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?