CHAPTER 1

Teaching Mathematics in The 21st Century

LOOKING AHEAD

The first decade of the new millennium is a good time to remember the past, consider the present, and plan for the future. The past century brought changes that transformed education. Some of the most drastic changes have come in mathematics education. At the turn of the last century, children studied arithmetic in the elementary grades. They did sums or long division on slates or, later, in lined paper tablets, and they memorized the times tables. Today, the third- and fourth-generation descendents of those schoolchildren log onto the Internet for information about fractals and Fibonacci numbers. In class they work with manipulatives and study economic concepts such as supply and demand; they even personally interact with astronauts as they conduct experiments on space shuttles

In this chapter we will look at some of the factors that brought about these changes and how the changes are working together to reconstruct or remake mathematics education for the 21st century. Building a consensus and setting standards for mathematics education have proceeded in the context of national debates over curriculum, evaluation, and professional development-debates sometimes called the "math wars." From these "wars" have emerged goals and documents such as *Principles and Standards for School Mathematics* and Project 2061 as well as standards at the state and local levels. Mathematics educators may hold differing ideas about methods, curriculum, content, and even criteria for excellence. Most, however, share a commitment to increasing the scope, accessibility, and excellence of mathematics education in the 21st century.

Being a Teacher in the 21st Century

Fifty years ago in a small-town classroom, a teacher with a vision for the future told her students, "By the end of this century you may be living in automatic houses where everything from cooking to cleaning is done for you. You'll probably wear disposable clothes. You might even vacation on the moon or work on Mars." What she predicted hasn't happened yet, although we have taken the first steps toward interplanetary travel; in Canada there are experimental "smart" towns; and our refrigerators may soon be able to talk to us about souring milk or needed items for our grocery lists. The teacher wasn't totally accurate but she was clairvoyant—a clear seer. What she saw clearly and what she helped her students see was that the future was filled with wonderful possibilities if only they would "dream big"—set high goals, work to make dreams happen, and believe in themselves.

"Dreaming big" will be a prerequisite for teachers in the 21st Century. Never before has so much been expected of us, and never before has so much depended upon us.

A hundred years ago a teacher had succeeded if she taught a few things to the many and many things to the few. Those who fell behind or dropped out could always find jobs on the farms and in the factories. Their livelihood didn't depend upon "school" learning; learning outside the school provided enough to get by in their agrarian, blue-collar world.

All of that has changed. Few can live on the wages from semi- or unskilled labor. It's brains, not brawn, that are needed to survive in the information age, and brains need more than basic training to function at their best; they need knowledge and understanding.

Beyond Shop-and-Yard Mathematics

The challenge for teachers and their students to "dream big" is perhaps greatest in mathematics education. In the first half of the 20th Century, curriculum development emphasized shop-and-yard skills. Prompted by the idea of functionalism (education you can use), some educators focused on identifying minimal competencies needed to perform different jobs: dollars-and-cents math for clerking, feet-and-inches math for carpentry, measuring-cups-and-spoons math for cooks and homemakers.

The changing needs of a changing world have made this restrictive view not only obsolete but also dangerous. The student who knows no more than shop-and-yard mathematics risks being left behind in a job market that increasingly emphasizes technology and information systems; risks being left out of the national and international discourses about economics, politics, science, and health care; risks, in short, the handicap of mathematical illiteracy. (See Figures 1.1 and 1.2 for examples of the important mathematical topics being tackled by fifth and ever second graders today).

In Step with the New Mathematical Literacy

The National Council of Teachers of Mathematics (NCTM) has identified five imperatives or needs for all students (NCTM 1998, 45-46).

- Become mathematical problem solvers.
- Communicate knowledge.

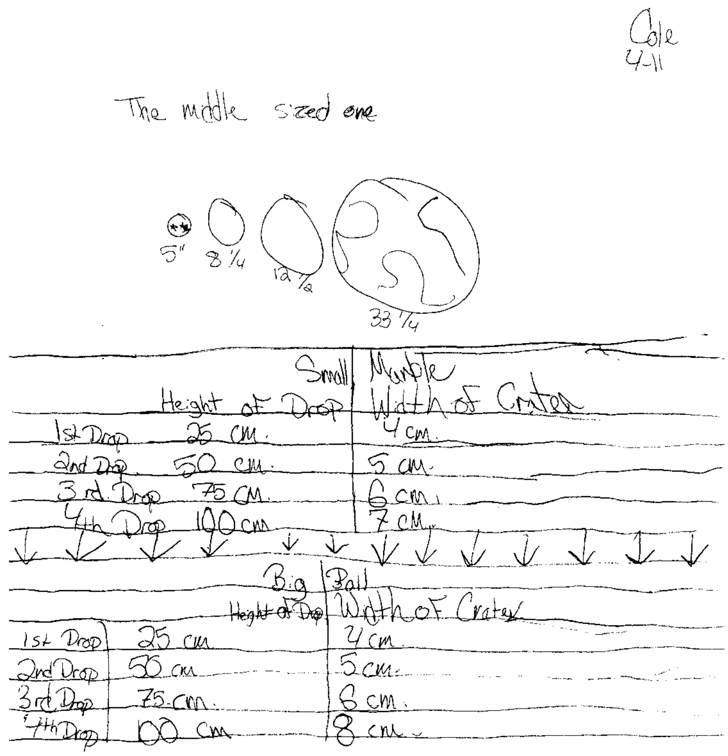


Figure 1.1 Tackling the information age task of data collection, fifth graders collect data on crater sizes made by dropping different object from different heights.

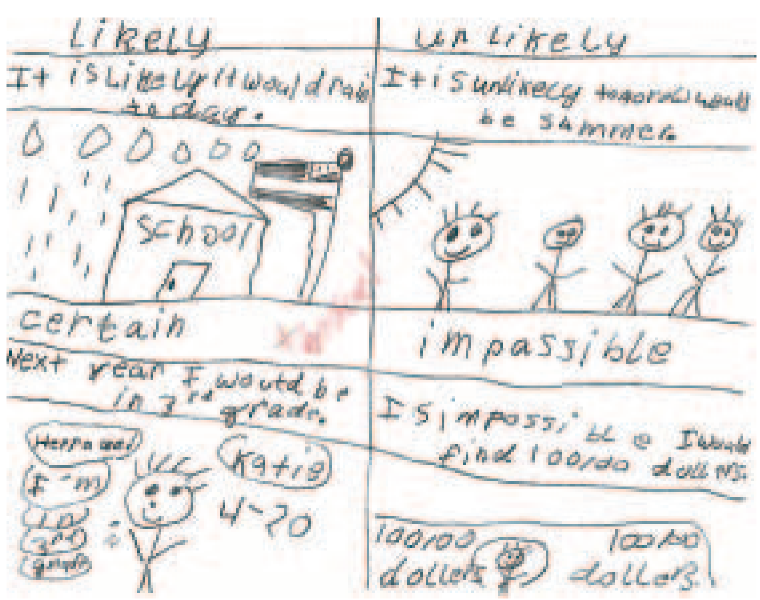


Figure 1.2 Second graders explore the language of probability.

- Reason mathematically.
- Learn to value mathematics.
- Become confident in one's ability to do mathematics.

Teachers Self-Inventory

1. What can I hope to accomplish as a teacher in the 21st century?
2. Am I ready for the challenge of teaching everything to everyone?
3. Am I ready to dream big—to aim for excellence as a teacher of mathematics?
4. Can I instill the ability to dream big in my students—excellence in learning mathematics?
5. Can I go beyond teaching basic skills and model the joy and beauty of mathematics?
6. Do I appreciate mathematics myself?
7. Do I really believe—not just think, but believe— that everyone can learn to reason mathematically?
8. Do I feel confident in my mathematical ability?
9. Have my own mathematical abilities been developed beyond the level of performing basic procedures?
10. Do I understand and can I interpret for my students the mathematical worlds that surround us?

Figure 1.3 Teacher's self-inventory.

Topics, Issues, and Explorations

As teachers, we model behaviors and attitudes for our students—some consciously, some unconsciously. What kinds of models for teaching mathematics have you had? Which models do you want to be like? Which models do you not want to be like?

These are in effect he cornerstones of the new mathematical literacy—what's needed to survive and thrive in the next century.

Meeting these imperatives calls for more than hard work and good intentions; it calls for belief—belief in our own abilities to teach and belief in our students' abilities to learn. The Teachers' Self-Inventory in Figure 1.3 suggests some things to think and talk about as you set your goals for professional development and growth.

Changing Views About Who Should Learn Mathematics

During much of the 20th Century, opportunities to study mathematics were often unequal. All students studied arithmetic, but only the college-bound elite tackled mathematics. The exclusionary process frequently targeted women and minorities, creating a hierarchy of expectations and opportunities that pushed children in one direction or the other from the earliest grades—often without the children's or their teachers' realizing what was happening.

Research data show that millions of people have been victims of false assumptions about who has the ability to master mathematics.

> These assumptions become self-fulfilling expectations, which ultimately undermine the self-concepts of female students, impoverished students and students of color.
>
> The single most important change required involves a national consciousness raising. Teachers, parents, and the students themselves must recognize that virtually every child has the capacity to master mathematics. . This is true for females as well as for males, for poverty-stricken students as well as those from more affluent backgrounds, and for persons of every ethnicity
>
> *(Drew 1996, 2-3).*

NCTM took a significant step toward "consciousness-raising" by recommending the Standards for all students. Instead of tiering objectives—more mathematics for the college bound, less for prospective trade school students, and almost none for at-risk students—the Council asks for more mathematics—more emphasis, more complexity, more challenging goals and objectives—for *all* students.

Tradition and Myths

But, you might ask, is this wise? Are we ignoring meaningful differences in aptitude in the interest of equity and fair play? Won't expectations be lowered and students who excel in mathematics, shortchanged? Behind these questions lie some of the most damaging of the math education myths:

- Mathematics is a subject so demanding that few can hope to understand it.
- Equal treatment to one group somehow subtracts something from another.
- Mathematics education should be layered—advanced concepts for the few, basic concepts for the many, math facts for the rest.

It is a mark of the power of tradition that myths such as these continue to fuel the national debate over reforming mathematics curricula. Look outside our own country and the arbitrary nature of some of our curriculum "truths" becomes apparent. In China, where far fewer resources can be devoted to education, almost everyone learns advanced mathematics. "It is assumed," writes David Drew, "that everyone can master advanced concepts and everyone is expected to do so" (1996, 9). Robert Reich, in *The Work of Nations*, says, "Japan's greatest educational success has been to assure than even its slowest learners achieve a relatively high level of proficiency" (1991, 228). In the Trends in International Mathematics and Science Study (TIMSS), the United States has been consistently outperformed by third world countries—countries whose "slowest learners" might have been suspected of "holding back" the majority if they had been studying in American classrooms (Gonzales et al. 2004; U.S. Department of Education [USDE] 1998).

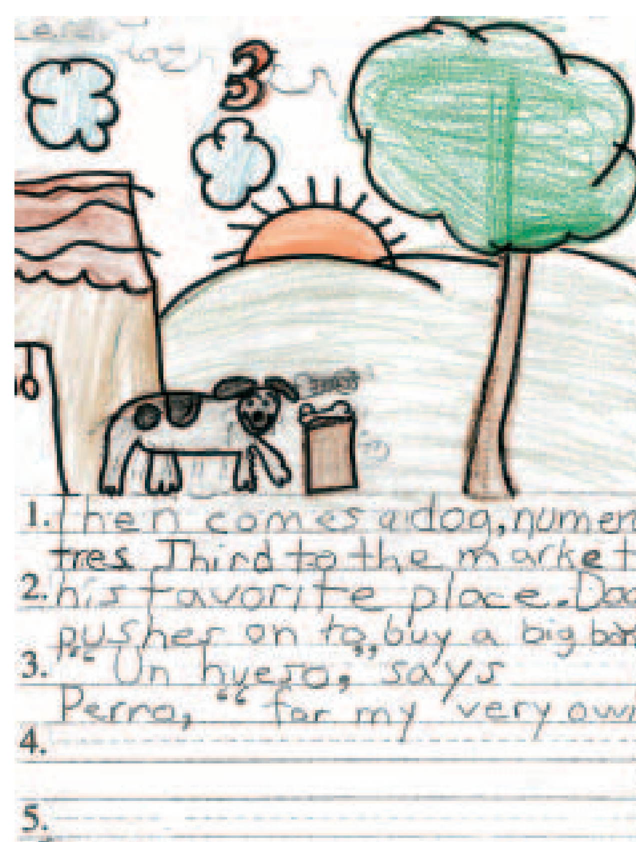


Figure 1.4 Mixing English and Spanish in bilingual math classes.

Equity Reforms

In the United States recent reforms probably began to affect performance in measurable ways in the 1990s (see Figure 1.4 for an example of one reform, a bilingual math class). However, while test

Topics, Issues, and Explorations

How do you see knowledge of mathematics—as an abundant or a scarce resource? If you select the former, what are some of the issues involved in the teaching challenge Drew mentions? If the latter, how would you decide who should be given the opportunities to learn the higher levels of mathematics?

Activities CD

For an activity related to a bilingual lesson in Spanish and English, see Activity 20 on the CD-ROM that accompanies this text.

scores for women and minorities have risen significantly, the performance of students who traditionally do well on achievement tests has neither declined nor fallen behind (National Center for Education Statistics 1998, 72–73; Stevens 2003). And study after study shows the benefits of mainstreaming and integrating rather than separating students (see also West 1991).

We have two choices as teachers, Drew writes: we can assume that

1. virtually everyone can master the material and the challenge is to present it in a manner that allows them to do so, or
2. the material is tough and only a few of the best and brightest will be able to learn it

(9)

The assumptions we make will not only affect our classroom behaviors and expectations but also students' perceptions about their own abilities and potential to learn mathematics.

Changing Views About How Students Learn

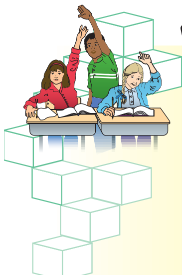
Perhaps the most dramatic changes in school mathematics during the 20th century were in the way children study mathematics. Consider the classroom scenarios described in the Windows on Learning feature.

The children in the first scenario are learning what one writer calls "muscle" or "muscular" mathematics (Betz 1948, 203). They exercise their mental muscles with repetitions intended to make responses automatic, without thought. The teacher is the center of the class, in control of learning as well as behavior. The environment of the class is disciplined and quiet. The consequences of failure are immediate and devastating—public discussion of errors with a pejorative thrust.

The mathematics activity in the second scenario reflects some changes in our perspective, both about learning and about student-teacher roles in the learning process. Instead of drilling and memorizing facts, these children explore ideas like scientists, with a problem to solve, materials to experiment with, and a spirit of inquiry. This is dynamic instead of passive or static learning, and the children rather than a teacher direct and shape the process. Multiple rather than single outcomes are not only possible but also encouraged. The activity is open ended; the learning cooperative. The small group is a learning, team in which the flow of ideas is unstructured and spontaneous and the possibilities, are limitless.

Changing Views About What Should be Learned

In 1994, NCTM changed the name of its journal for elementary teaching from *The Arithmetic Teacher* to *Teaching Children Mathematics*. The change marked a major transition in the way we think and talk about mathematics learning in the elementary grades as well as changes in the content itself. Today mathematics is



Windows on Learning

"Two Times Two is Four"

The year was 1954; the place, Mrs. Taylor's third-grade classroom at Briscoe Elementary School.

The students sat quietly, their hands folded in front of them, at desks lined up in five neat rows, with six desks to a row. The desks were all filled. The first group of baby boomers were entering the public schools, and space and teachers were at a premium.

Mrs. Taylor stood before the class at a chalkboard. She had just finished correcting the work of six students who were called to the board to do multiplication problems involving two-digit numbers. The exercise had not gone well, and Mrs. Taylor was frustrated.

"Billy, you multiplied 33 times 33 and got 66. You know that can't be right; 3 plus 3 is 6, so 3 times 3 can't be the same thing.

"Suzanne, you say 15 times 55 is 770. How can that be if 5 times 5 is 25? The number can't end with a 0.

"What's happening, class, is we are forgetting our times tables." She picked up a stick and pointed to a chart above the chalkboard.

"Everybody stand."

The students slipped quickly out of their desks and stood with their hands at their sides and eyes on the chart. Everyone was careful not to look at Billy and Suzanne, who were red faced and embarrassed.

"All right, everyone, together now on the count of three."

Mrs. Powell tapped the chart three times with her stick, and the children began to chant, "One times 1 is 1, 2 times 2 is 4. . . ."

"Two Times Two Is Square"

The year was 2004 (half a century later). At Lowell Elementary School, in Room 123, fourth graders were experimenting with multiplying numbers by themselves.

Tino, Shelley, Angie, and Letitia were working in a small group around a circular table. In front of them were manipulatives, some colored blocks, various tools for measuring, and scratch paper and pencils for sketching and trying out ideas.

"What are we supposed to be doing?" Tino asked.

Letitia consulted a list of objectives in her three-ring journal.

"We've done the map activity and the frequency count. That leaves multiplying and shapes for this week. Which one would you rather do?"

The group decided to work on multiplying and leave shapes for the next day. The assignment was simple: find out what happens when you multiply numbers by themselves.

"That's easy," Angie said. "It's like adding them up over and over."

"Like this," Hussein agreed and began to arrange blocks on the table in front of them, two sets of two blocks, one on top of the other, for 2 times 2; three sets of three blocks for 3 times 3.

Shelley sat watching Hussein line up the blocks. She didn't say anything, but she had a feeling he was missing something by lining the blocks up.

Meanwhile, Tino was verbalizing what Hussein was doing. "You put two sets of two together and get 4, three sets of three and get 9, four sets of four and get 16, five sets of five and get 25."

"Hey, everybody, look at this," Angie said, looking up from the pad where she had been doodling. "If you write all the numbers down, 1's odd, 4's even, 9's odd, 16's even, 25's odd."

Letitia was working with cuisenaire rods, arranging and rearranging them as she looked for patterns.

Then Shelley reached a tentative hand toward the blocks in Hussein's 2 times 2 line. "I think these would look better like this," she said and quickly rearranged the blocks into a square.

Hussein saw what she was doing and joined in.

"Does it do that every time?" Angie stopped doodling to ask.

"I don't know. I think so," Shelley said and kept on moving blocks. Finally, all of Hussein's blocks had been arranged into squares.

"It happens every time. The blocks make a square," Hussein observed.

"So when you multiply a number by itself, you get a square," Letitia summarized.

Later when the group discussed the activity with one of the class's team teachers, Ms. Lee, she suggested they see what the math software the class used had to say about squaring. The computer software reinforced the block arranging Shelley and Hussein had done with graphics of squares being multiplied into larger and larger squares. It also showed them how to represent the squaring process in math language with a superscript^2.

a foundational discipline. It provides tools and ways of thinking that impact learning across the curriculum.

Some factors that influenced the changing mathematics curriculum in the 20th Century included changes in our economic and social worlds, historical events and trends, and new developments in technology and science.

Tying the Curriculum to Mental Age and Social Utility

Early attempts to design a mathematics curriculum focused on matching content to students' mental age and, therefore, readiness to learn. For example, in the 1920s, school administrators collected survey data to tie arithmetic topics to children's "mental ages." They used their correlations to sequence the curriculum, "delaying" introduction of many topics such as multiplication and division because of students' supposed "mental" unreadiness (Washburne 1931, 210, 230–31). Readiness, according to these administrators, could be determined by a combination of intelligence and achievement tests, which would allow teachers to "ability-group" students or individualize instruction. They concluded that arithmetic was too hard for most elementary school students and should be taught in junior high or high school instead (see Brownell 1938, 495-508, for a critique).

Just as the Great Depression turned nations inward, the social utilitarians of the mid-20th Century advocated a short-range rather than a long-range view for the mathematics curriculum. Guy Wilson, one of the movement's leading proponents, believed the schools should teach the skills required to do adult jobs. In 1948 he wrote:

> The proper basis for functional arithmetic is the social utility theory. This theory posits (1) that the chief purpose of the school is to equip the child for life, life as a child, life as an adult, and (2) that the skills, knowledges, and appreciations should receive attention in school somewhat proportional to usefulness in life
>
> *(321)*

Wilson (1948) identified basic arithmetic facts needed by the majority of workers and used them to calculate what he called "the drill load of arithmetic"—the facts and skills for a drill mastery program in which "[o]nly success is wanted and only perfect scores" (327, 335). Students, according to Wilson, should memorize 100 primary facts each for addition, subtraciton, and multiplication:

> (1) *Addition*—100 primary facts, 300 related decade facts to $39 + 9$, 80 other facts for carrying in multiplication to 9×9. . . . Whole numbers only. . . .
> (2) *Subtraction*—100 primary facts, all process difficulties. . .whole numbers only. . . .
> (3) *Multiplication*—100 primary facts, all process difficulties, whole numbers only. . . .
> (4) *Division*—emphasis on long division. . . .
> (5) *Common fractions*—. . .halves and quarters, thirds, possibly attention to eighths and twelfths separately.
>
> *(327-28.)*

Wilson recommended little or no work with decimals since "decimals represent specialized figuring learned on the job" (1948, 329). Measures, percentages, geometry, and algebra were relegated for the most part to what he called "appreciation" study—studies undertaken for "fun" and used to "lure" the brightest students forward. Wilson also argued that the metric system should not be taught because English measures were more convenient: "The housewife, even in a metric country, wants a pound of butter" (1948, 327). Light years, parsecs, measurements related to the electronic age—should they be taught? "No, of course not," wrote Wilson. "The numbers using [them] are too few" (1948, 337).

The social utility argument continues to influence curriculum choices. As recently as the 1980s the National Center for Research in Vocational Education published a series called *Math on the Job*, with special kinds of numbers for the grain farmer, mechanic, clerk, machinist, cashier, and so forth.

Responding to a Bigger World

Even as the utilitarians were urging a reduced mathematics curriculum, others were calling for expansion. World War II had shown Americans a bigger world—a world where Swiss students studied calculus in high school, where scientific breakthroughs were needed, not just to win but to survive. The Commission on Post War Plans called for more, not less, mathematics in education. In 1947 the President's Commission on Higher Education proposed increasing college enrollments drastically for a minimum of 4 million by 1960—a change that would require a college-track mathematics curriculum for millions. By the time the Soviet Union launched *Sputnik* in 1957 and galvanized public opinion for the space race, educators were already experimenting with new mathematics curricula.

The "new math" as it was popularly called, emphasized mathematics structure. Students studied sets, number systems, different number bases, and number sentences. Teachers guided children to discover concepts rather than lecturing about them. While many ideas of the new math had merit, application may have been flawed. Textbooks were often hard to read and overly formal. Many parents complained that they could not understand their children's homework.

In the meantime, the social revolution of the sixties and seventies flooded colleges with students—many from backgrounds and groups that traditionally had not attended college. To what extent this new college population affected test scores remains unclear, but between 1963 and 1975 SAT scores declined, leading to several major concerns for the mathematics curriculum in the final decades of the century, including how to

- upgrade the curriculum to match the demands of an increasingly technological society,
- balance student needs with the needs of society and of mathematics itself, and
- teach the expanded curriculum to all of the students.

There were no easy answers. A back-to-the-basics movement called for a return to traditional mathematics—teacher lectures, drills, and tests. But many argued that traditional approaches had worked for no more than 5% to 15% of the students; what was needed was a challenging mathematics curriculum that prepared every student to think mathematically—to develop the foundations in mathematical reasoning, concepts, and tools needed for advanced mathematics education as well as enlightened living in the age of technology.

Topics, Issues, and Explorations

An effective curriculum is multi-dimensional. It responds to the needs of society, the needs of the individual, and the needs of the subject. Think about the changes in the mathematics curriculum in the 20th Century. Which changes do you think reflected concerns about which needs? Which changes seem most worthwhile or least worthwhile?

The National Council of Supervisors of Mathematics (NCSM) responded with a list of basic skills (1977) and later with "Essential Mathematics for the Twenty-first Century "(1989). NCTM did the same, producing an *Agenda for Action* in 1980, the first version of the *Curriculum and Evaluation Standards* in 1989, and now the Standards 2000 document, *Principles and Standards for School Mathematics*, compiled with the input of thousands of mathematics teachers responding over the World Wide Web. Some major points of consensus between the NCSM and the NCTM recommendations include the following:

- that all students benefit from a challenging mathematics curriculum;
- that mathematics reasoning and higher-order thinking skills should be integral to the curriculum;
- that problem solving should be a priority;
- that algebraic thinking, geometry, statistics and probability are essential rather than add-on skills;
- that the emphasis in computation should be on meaning and patterns;
- that communication of mathematical ideas in a variety of ways (oral, written, symbolic language, everyday language) is critical to the learning process;
- that students need opportunities to explore and apply mathematics in hands-on and real-life activities.

Building Consensus and Setting Standards

Changes in curriculum and pedagogy are not like changes in the seasons, though they may be just as inevitable. Few of the changes described in the previous sections have come smoothly or without controversy. In his 1998 address, "The State of Mathematics Education: Building a Strong Foundation for the 21st Century," then–secretary of education Richard W. Riley called for a "ceasefire" in the "math wars" about "how mathematics is taught and what mathematics should be taught." "We need," he told the meeting of the American Mathematical Society and the Mathematical Association of America,

> to bring an end to the shortsighted, politicized, and harmful bickering over the teaching and learning of mathematics. I will tell you that if we continue down this road of infighting, we will only negate the gains we have already made—and the real losers will be the students of America.
>
> I hope each of you will take the responsibility to bring an end to these battles, to begin to break down stereotypes, and make the importance of mathematics for our nation clear so that all teachers teach better mathematics *and* teach mathematics better.

Riley appealed for "civil discourse" and openness to change. The controversy reached mud-slinging levels in the 1990s, with reformers accused of teaching "fuzzy math" or "placebo math" or "dumbing down to promote classroom equality" (Mathematically Correct 1997; Leo 1997, 14). But reforming the mathematics curriculum has always been a stormy process. In 1948 Willian Betz complained, "For nearly six decades we have had unceasing efforts at reform in mathematics," and, he pointed out, "milestones in this epic struggle" go back to 1892 (197). He wrote, "We have looked at a picture which is no doubt perfectly familiar to every experienced teacher of mathematics. It is that of a battle between two sharply contrasting positions regarding the educational role of mathematics (1998, 205). In the National Society for the Study of Education's 1970 yearbook, *Mathematics Education*, Lee Shulman, citing articles published in 1930, 1935, and 1941, says they "can almost read as a history of controversies, cease-fires, and temporary truces..." (23).

Although the tone of the controversies may at times have sunk below the levels of civil discourse urged by Secretary Riley, the controversies themselves may not be unproductive. In fact, even the emotionally charged skirmishes may serve a purpose since they tend to involve the public in the dialogue about reform.

Nonetheless, if consensus among mathematicians is neither clear nor stable, is it worthwhile to set standards, and can the standards set be worthwhile? If we think of standards as commandments engraved in stone, the answer may be no. However, if we accept setting standards as an ongoing and open-ended process, the answer is yes. According to *Webster's New World Dictionary*, the word *standard* originally meant "a standing place." The meaning has grown to include flags or banners that symbolize nations, causes, or movements; levels of attainment set as benchmarks; and even foundation supports. Finding out where we stand and establishing goals, benchmarks, and supporting structures for those ideas have all been part of the standard-setting process—or process*es* since efforts to set standards are ongoing at state and national levels and for a variety of curriculum and development areas.

Although driven by a dialogue that has ranged in tone from the rational to the acrimonious, these standards-setting processes have succeeded at several levels. First, they have generated research and ideas that have disrupted the status quo, jarring entrenched assumptions about mathematics education and opening the way for new concepts and methods. Second, they have focused attention on critical issues, such as equity and technology in teaching mathematics. And third, they have generated public interest and involvement at unprecedented levels. When in our history has mathematics in the schools been discussed and debated with greater intensity and urgency? Making mathematics education a national issue may have been one positive outcome of the math wars. Mathematics, like science, occurs in a social context (see Drew 1996, 17). Engaging society in the debate over what is taught and how it is taught ensures that reform takes place within rather than outside the social context and remains responsive to the needs and demands of those most directly affected by the changes.

Topics, Issues, and Explorations

Identify and explain one concept, content area, or process that you believe should be learned during a specific grade. Share your ideas in a group or class. How much agreement or disagreement do you find?

National Standards for Mathematics Education

In *Goals 2000: Educate America Act*, Congress proposed in 1994 "a national framework for education reform" and called for "the development and adoption of a voluntary national system of skill standards and certification." (See Figure 1.5)."
The act responded in part to efforts already under way by professional groups

The 1994 *Goals 2000: Educate America Act* challenged schools both to achieve and to compete.

(A) By the year 2000, United States students will be first in the world of mathematics and science achievement.
(B) The objectives for this goal are that–

(i) mathematics and science education, including the metric system of measurement, will be strengthened throughout the system, especially in the early grades;

(ii) the number of teachers with a substantive background in mathematics and science, including the metric system of measurement, will increase by 50 percent and

(iii) the number of United States undergraduate and graduate students, especially women and minorities who complete degrees in mathematics, science, and engineering will increase significantly. (Educate America Act of 1994; see also National Education Goals Panel 1995).

Figure 1.5 Setting national goals for mathematics education.

such as NCTM and the American Association for the Advancement of Science (AAAS).

Underlying these goals and objectives are several basic assumptions: that having an informed citizenry is essential to national security and productivity; that being informed entails higher levels of achievement in mathematics and science; that "being first" is a desirable and feasible outcome; that a nation that exemplifies diversity can set common standards and achieve common goals in mathematics education.

NCTM's Principles and Standards 2000

Principles and Standards for School Mathematics (2000) integrates areas covered by three earlier Standards publications: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995). The purpose of the Standards 2000 document is ambitious and broad: "to set forth a comprehensive and coherent set of goals for mathematics for all students from prekindergarten through grade 12 that will orient curricula, teaching, and assessment efforts during the next decades" (NCTM 2000, 6). To this end, the document proposes a vision, principles, and standards to be applied across four grade bands: pre-kindergarten through grade 2, grades 3-5, grades 6-8, and grades 9-12. The vision is both idealistic and far-reaching:

NCTM Vision for School Mathematics*

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

(NCTM 2000, 3)

NCTM's Vision for School Mathematics assumes both the importance of knowing mathematics in the 21st Century and the need to continually improve mathematics education to meet the challenges of a changing world (see Figure 1.6 for an example of a second grader's use of modern technology in a counting activity). Understanding and using mathematics is described as an essential underpinning of life, a part of our cultural heritage, and a prerequisite for success in the workplace. And providing all students with "the opportunity and the support to learn significant mathematics with depth and understanding"

Figure 1.6 Counting with computer graphics.

is linked to "the values of a just democratic system" and "its economic needs" (NCTM 2000, 5).

NCTM's Principles for School Mathematics are equally far-reaching:

NCTM Principles for School Mathematics

- **Equity:** Excellence in mathematics education requires equity—high expectations and strong support for all students.
- **Curriculum:** A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
- **Teaching:** Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- **Learning:** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- **Assessment:** Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- **Technology:** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

(NCTM 2000, 11)

Together with the Standards, these Principles comprise key components of NCTM's vision of high-quality mathematics education. The Principles are, in effect, ideals to live by—foundational ideas that influence curriculum and professional development on the larger scale as well as instructional decisions in the classroom on the smaller scale. The Standards are more like building materials. They outline mathematics content and processes for students to learn. Instead of the multiple standards of the 1989 document, *Principles and Standards for School Mathematics* proposes 10 standards that "specify the understanding, knowledge, and skills students should acquire from kindergarten to grade 12."

> The Content Standards—Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability—explicitly describe the content that students should learn. The Process Standards—Problem Solving, Reasoning and Proof, Communication, Connections, and Representation—highlight ways of acquiring and using content knowledge.
>
> *(NCTM 2000, 29)*

Each Standard entails goals that apply across all grades plus differing emphases for the grade bands (see Figure 1.7). For example, number and measurement are emphasized in the early grades, while later grades spend more instructional time on formal algebra and geometry. Arranging the curriculum into 10 standards that span the grades offers a coherent structure for an overall curriculum. Specific details are left to those who will apply and implement the ideas.

Topics, Issues, and Explorations

NCTM gives detailed "Expections" or specific objectives for each Content Standard by grade group. Study the Expectations in *Principles and Standards for School Mathematics* for the grade you are teaching or plan to teach. Do any surprise you? Discuss your reactions in small groups.

PROCESS STANDARDS

Standards Instructional programs from pre-kindergarten through grade 12 should enable all students to—

Problem Solving
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof
- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Connections
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

CONTENT STANDARDS

Standards Instructional programs from pre-kindergarten through grade 12 should enable all students to—

Number and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

Algebra
- Understand patterns, relations, and fractions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

Data Analysis and Probability
- Formulate questions that can be addressed with data and collect, organize, and display data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate ingerences and predictions that are based on data
- Understand and apply basic concepts of probability

Figure 1.7 NCTM Process and Content Standards (NCTM 2000, 392-403).

Project 2061, Science for All Americans

A parallel project to NCTM's Principles and Standards began in 1985, the date of the last visit of Halley's comet. Sponsored by AAAS, Project 2061 is named for the date when Halley's comet will return and assumes that children who were beginning school in 1985 will see a lifetime of changes in science and technology before the comet's return in 2061. To prepare them for these changes, Project 2061 proposes educational reforms akin to those promoted by NCTM. Culotta suggests seven major areas of commonality:

- Less memorization
- Involvement of teachers in the reform process
- Integration of disciplines and study
- Greater emphasis on hands-on activities
- Greater focus on listening to students' questions and ideas
- Connections between discipline and society
- Emphasis on the scientific process and how problems are solved

Project 2061 defines mathematics as "the science of patterns and relationships" and describes it as "the chief language of science" (AAAS 1989). In the project's "Design for Scientific Literacy," mathematics is included in most of the building blocks for a Project 2061 curriculum: "For purposes of general scientific literacy, it is important for students (1) to understand in what sense mathematics is the study of patterns and relationships, (2) to become familiar with some of those patterns and relationships, and (3) to learn to use them in daily life" (AAAS 1989).

In Project 2061's "Benchmarks for Scientific Literacy", as shown in Figure 1.8, specific educational objectives are outlined by grade, with an emphasis upon outcomes or "what students should know" and understand. The Benchmarks emphasize the importance of experiencing mathematics, of establishing connections between ideas and areas of inquiry, of "making multiple representations of the same idea and translating from one to another" (AAAS, 2000). Implicit in the various objectives are ties to development; for example, the emphasis in the early grades is on the specific, concrete, and immediate, with the gradual introduction of abstract ideas and "grand categories" in later grades. "Doing mathematics," like "doing science," is encouraged from the earliest grades, and mathematical inquiry leading to the valid development of mathematical ideas also starts in the earliest grades when children explore concrete objects to discover what they tell us and what they can be used to show about the world around them.

Overall, Project 2061 proposes specific educational objectives within a context of scientific values and attitudes, including attitudes about learning:

> Students in elementary school have a spontaneous interest in nature and numbers. Nevertheless, many students emerge from school fearing mathematics and disdaining school as too dull and too hard to learn. . . .
>
> It is within teachers' power to foster positive attitudes among their students. If they choose significant, accessible, and exciting topics in science and mathematics, if they feature teamwork as well as competition among students, if they focus on exploring and understanding more than the rote memorization of terms, and if they make sure all their students know they are expected to explore and learn and have their achievements acknowledged, then nearly all of those students will indeed learn. And in learning successfully students will learn the most important lesson of all—namely that they are able to do so.
>
> *(AAAS, 1998 chap. 12)>*

Topics, Issues, and Explorations

Discuss NCTM's Standards and Project 2061's Benchmarks for learning mathematics. How are they alike? How are they different? Which Standards and Benchmarks seem most important to you?

Kindergarten through Grade 2

By the end of the 2nd grade, students should know that:

- Circles, squares, triangles, and other shapes can be found in nature and in things that people build.
- Patterns can be made by putting different shapes together or taking them apart.
- Things move, or can be made to move, along straight, curved, circular, back-and-forth, and jagged paths.
- Numbers can be used to count any collection of things.
- Numbers and shapes can be used to tell about things.

Grades 3 through 5

By the end of the 5th grade, students should know that:

- Mathematics is the study of many kinds of patterns, including numbers and shapes and operations on them. Sometimes patterns are studied because they help to explain how the world works or how to solve practical problems, sometimes because they are interesting in themselves.
- Mathematical ideas can be represented concretely, graphically, and symbolically.
- Numbers and shapes—and operations on them—help to describe and predict things about the world around us.
- In using mathematics, choices have to be made about what operations will give the best results. Results should always be judged by whether they make sense and are useful.

Grades 6 through 8

By the end of the 8th grade, students should know that:

- Usually there is no one right way to solve a mathematical problem; different methods have different advantages and disadvantages.
- Logical connections can be found between different parts of mathematics.
- Mathematics is helpful in almost every kind of human endeavor—from laying bricks to prescribing medicine or drawing a face. In particular, mathematics has contributed to progress in science and technology for thousands of years and still continues to do so.
- Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone.

Figure 1.8 Project 2061 Benchmarks in Mathematics for the Elementary Grades through Middle School.
Source: AAS (2000).

State and Local Standards

Efforts to develop national standards have had a significant impact on mathematics education overall. For example, in 1996 the framework for the National Assessment of Educational Progress (NAEP) was revised to reflect NCTM curricular emphases and objectives (U.S. Department of Education [USDE] 1999, 2-3). National standards have also influenced the development of standards at the state and local levels. Some states have adapted the national standards to fit their own school districts' needs (see, for example, *Colorado Model Content Standards* 2005.) Others have created their own benchmarks and detail what students should

Topics, Issues, and Explorations

What standards has your state or district established for mathematics? The information may be available at your state or county Web site, or you can ask a school librarian for help. How do these standards compare to NCTM's Standards 2000 or to Project 2061?

know grade by grade. Georgia's performance-based standards are actually aligned with Japanese standards as well as the Georgia Criterion-Referenced Competency Tests (www.georgiastandards.org and www.glc.k12.ga.us/).

Meeting the Challenges of the 21st Century

The 20th Century began the process of reconstructing mathematics education. In 1900, according to a writer in NCTM's *first yearbook*, the purpose of teaching arithmetic had as much to do with discipline as curriculum. "It was felt that the subject should be hard in order to be valuable, and it sometimes looked as if it did not make so much difference to the school as to what a pupil studied so long as he hated it" (Smith 1926, 18-19). Responding to the period's rigid and often lifeless teaching methods and materials, the president of the American Mathematical Society, Eliakim Moore, appealed to teachers:

> Would it not be possible for the children in the grades to be trained in power of observation and experiment and reflection and deduction so that always their mathematics should be directly connected with matters of thoroughly concrete character? . . .
>
> The materials and mathematics should be enriched and vitalized. In particular, the grade teachers must make wiser use of the foundations furnished by the kindergarten. The drawing and paper folding must lead directly to systematic study of intuitional geometry, including the construction of models . . . with simple exercises in geometrical reasoning . . . . The children [should] be taught to represent, according to usual conventions, various familiar and interesting phenomena and study the properties of the phenomena in the pictures to know, for example, what concrete meaning attaches to the fact that a graph curve at a certain point is going down or going up or is horizontal
>
> *(45-46).*

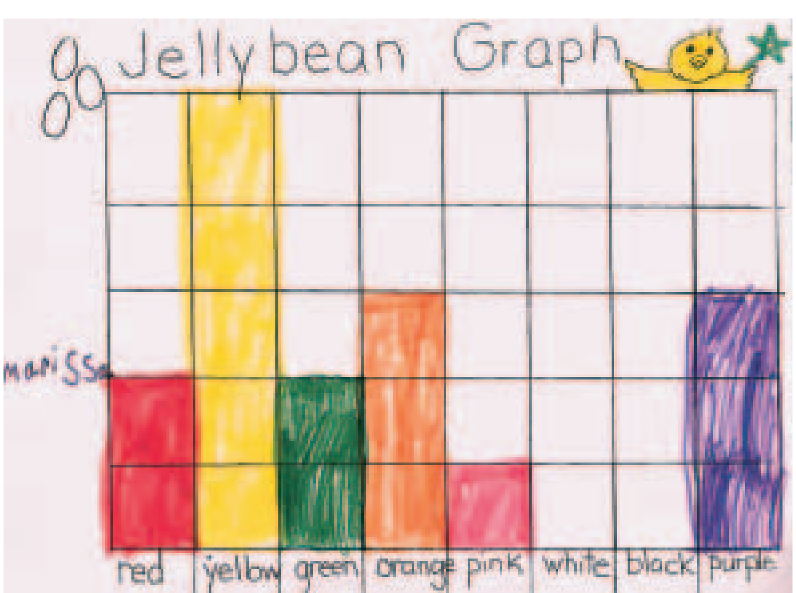


Figure 1.9 Children represent counting jelly beans with a bar graph.

Meeting the Challenges as a Nation

A hundred years later we can say that many elements of Moore's vision for learning mathematics are not only possible but also an accomplished fact. Hands-on, dynamic learning is becoming the norm in elementary classrooms (see Figure 1.9 for an example of a kindergartner's graphing of a hands-on counting activity). Technology has helped us enrich and vitalize the learning process with interactive learning experiences such as the National Center for Education Statistics' Students' Classroom (see the Math and Technology Feature, "Explore Your Math Knowledge"). Increasingly, lessons emphasize understanding and context and deemphasize rote memorization of isolated facts and procedures. The elementary curriculum is no longer limited to arithmetic but includes geometry, algebraic thinking, and mathematical reasoning that were once considered too abstract for children.

Evidence is mounting that these approaches are working and working well. After decades of declining test scores and public alarm about deficiencies, the trends seem to be reversing as shown in the graph in Figure 1.10. From 1990 to 2005, the National Assessment of Educational Progress (NAEP), the nation's report card, showed steady gains (2003; Perie, Grigg, and Dion 2005). SAT and ACT mathematics scores are up. The 2003 Trends in International Mathematics and Science Study (TIMSS) showed both U. S. fourth- and eighth-graders scoring above the international average in mathematics and science (Gonzales et al. 2004; NCES 2005; USDE 1997

Does this mean that the goals and objectives proposed by the *Educate America Act* have been reached? In the first decade of the 21st Century, is the U. S. first in the world in mathematics and science achievement? Perhaps yes, perhaps no. If being first is measured by achievements in the world of science and mathematics, the U. S. could stand at the top. If we look (as many in the national media do) at test scores, our position is less clear.

Although the results of the 2003 TIMSS placed U. S. fourth- and eighth graders above the international averages, U. S. fourth-graders were outperformed by students in 11 countries and U. S. eighth-graders by students in 9 countries. Students in four Asian countries—Chinese Taipei, Hong Kong SAR, Japan, and Singapore—outperformed both U. S. fourth- and eighth-graders (Plisko 2004). In addition the 2003 Program for International Student Assessment (PISA) placed U. S. 15-year-olds below the international average for both mathematical and scientific literacy (Lemke et al. 2004). Data collected for the NAEP in the 1990s showed no significant improvement in elementary or middle school teachers' preparation to teach mathematics (Hawkins, Stancavage, and Dossey et al. 1998). And the shortage of qualified mathematics teachers continues to grow, and women and minorities continue to be underrepresented in mathematics (Seymour 1995a, 1995b; Chaddock 1998).

Nonetheless, progress is being made. In the 1991 International Assessment of Educational Progress (IAEP), U. S. elementary school students scored below rather than above the international average (USDE 1997). Middle school students' performance improved significantly since the 1999 TIMSS. Moreover, data from NAEP show positive linear trends or overall increases in mathematics performance at all age levels tested from 1990 to 2003,

Math and Technology

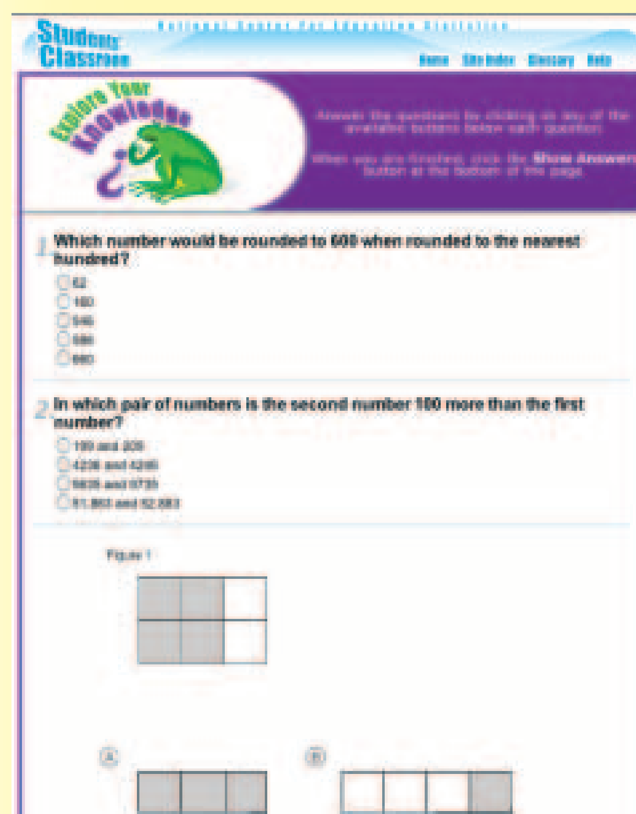
Explore Your Math Knowledge

The National Center for Education Statistics (NCES) has developed a Students' Classroom with activities, games, and learning experiences to encourage mathematics learning. The Web site is http://nces.ed.gov/nceskids/eyk/index.asp?flash=false.

The Explore Your Knowledge activity features questions from national tests such as TIMSS. Students respond to the questions and then check their answers. The activity tracks the number of correct answers and prompts students to "Try Again?"

Activity

Visit the NCES Students' Classroom. Explore some of the math questions, responding correctly and incorrectly. Discuss the various resources available at the site, and brainstorm ways to use the activities in the classroom, including the possibilities for using the questions to help students prepare for various national tests.



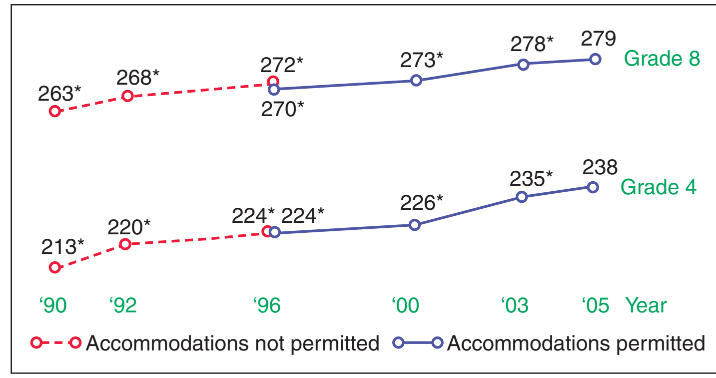
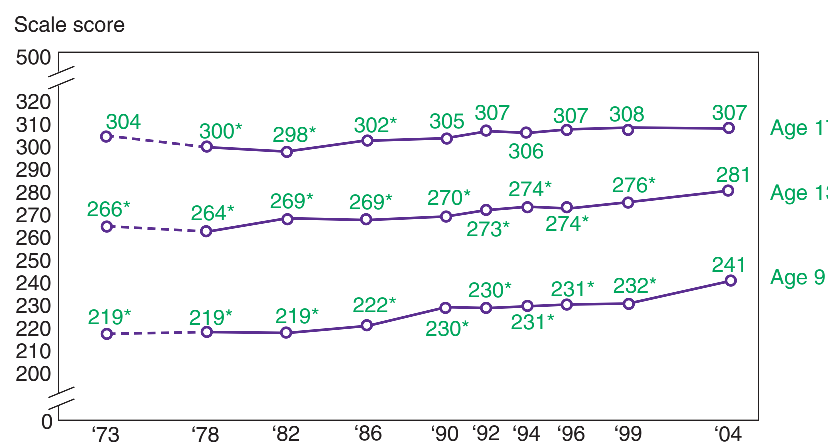


Figure 1.10 NAEP 1990-2005 trends chart

Source: U.S. Department of Education, Institute of Education Science, National Center for Education Statistics. National Assessment of Educational Progress (NAEP), various years, 1990–2005 Mathematics Assessment.



- **Nine-year-olds**. The average mathematics score of 241 was higher in 2004 than in any pervious assessment year.
- **Thirteen-year-olds**. The average score in 2004 was higher than in any other assessment year.
- **Seventeen-year-olds**. The average score in 2004 did not show a significant change when compared to the score in either 1973 or 1999.

Figure 1.11 NAEP 1973-2004 trends chart: National trends in mathematics by average scale scores.

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1973-2004 Long-Term Trend Mathematics Assessments.

continuing a positive trend begun in 1973 (see Figure 1.11; Perie et al. 2005; Perie and Moran 2005; USDE 2003, 1).

Comparisons of average scores in 1990 and 2005 show that the number of both fourth- and eighth-graders performing at or above the NAEP mathematics performance levels increased significantly (Perie et al. 2005, 1). The percentage of fourth graders who can perform basic numerical operations (adding, subtraction, multiplying, and dividing with whole numbers) and solve one-step problems more than doubled from the 1970s to 2004 (20% to

42%). The percentage of eighth graders performing at or above this level increased from 65% to 83% (Perie and Moran 2005). NAEP trends since the 1970s suggest a closing of the gender and race gaps at the fourth and eighth grade (USDE 1996, 1997, 1999, 2000; Perie and Moran 2005). Since 1982 the percentage of Hispanics and American Indians/Alaskan Natives taking mathematics courses beyond the basics more than doubled, and proficiency scores on the NAEP show steady improvement for all ethnic and racial groups (National Center for Education Statistics 1997, 1999; USDE 2000; Plisko 2003; Perie et al. 2005; Perie and Moran 2005; see the Math and Technology feature for Web sites devoted to NAEP, TIMSS, and other tests).

Mathematics education continues to face major challenges as we begin the new millennium:

Challenge 1: Building a national consensus about the value and accessibility of a challenging mathematics education for everyone

Challenge 2: Building a professional consensus about teaching and learning mathematics

Challenge 3: Continuing the reconstruction of mathematics education—

- reconstructing our views of mathematics—how we look at and think about mathematics;
- reconstructing our views of education—how we see our roles as educators, our students' roles, our teaching goals and outcomes;
- reconstructing assessment—developing and interpreting new tools that let us look beyond right or wrong answers and evaluate problem-solving strategies and mathematical thinking
- training teachers who are committed to the ideals and ready to face the challenges of teaching meaningful mathematics to all students

Meeting the Challenges in the Classroom

What kind of teacher is needed to implement reforms in mathematics education? What kind of teaching is needed to make reformers' vision for learning mathematics in the 21st Century a reality? Val Penniman, the teacher profiled in this chapter, discovered that change meant reinventing herself both as a math learner and as a math teacher. Although Val had had negative learning experiences in mathematics, she considered herself to be a good math teacher. She had memorized the scope and plan of her class's textbook and felt confident about her traditional method of teaching. The change for Val

Topics, Issues, and Explorations

According to researchers, a difference between the U. S. and most competing countries, including those that participated in the TIMSS study, is our lack of a national curriculum. What do you think of the idea of a national curriculum? Do we need a national curriculum in mathematics? How would it differ from current practice? What might we lose and what might we gain?

Math and Technology

You can find reports on TIMSS and NAEP as well as the latest research analyses and commentary on the Web.

http://nces.ed.gov/nationasreportcard/

This site provides updates on the National Assessment of Educational Progress, including study data, sample test items, and information about current tests.

http://nces.ed.gov/programs/coe

The National Center for Education Statistics prepares an annual report entitled *The Condition of Education.* The report includes a section on learner outcomes with specific data about mathematics proficiency and participation.

http://nces.ed.gov/timss/

Reports and analyses from the 2003 TIMSS are available at this site as well as information about earlier studies, including the 1999 TIMSS-Repeat, which focused on the progress of eighth graders in improving performances.

http://www/ed/gov/legislation/GOALS2000/TheAct/index.html

The text of the *Educate America Act* outlines plans for American education in the future. The act's intent is "to improve learning and teaching by providing a national framework for education reform.".

http:/www.ed.gov/ndb/overview/intro/4pillars.html

The website for the No Child Left Behind Act provides not only information about the act itself but also about the conditions it attempts to address and about successful teaching methods.

Preservice teachers develop materials and practice using hands-on methods for teaching math.

came with a SummerMath for Teachers program at Mount Holyoke College in Amherst, Massachusetts. She learned a problem-solving approach in workshop classes that modeled effective methods for hands-on, collaborative learning. The result was a new approach to learning and teaching mathematics that has paid off in the classroom with high test scores and excitement about learning.

Penniman's experience underscores the need for change in both the way teachers teach mathematics and the way they themselves are prepared to teach mathematics. NCTM's *Professional Standards for Teaching Mathematics* emphasizes teaching that helps students develop mathematical power. The six standards for teaching and six standards for professional development present a view of teaching and teacher education that focuses on students and is flexible and adaptive rather than formulaic (see Figure 1.12).

While the Standards 2000 document does not spell out new standards for teaching and teacher development, it does illustrate effective practices. Each segment devoted to process standards includes a discussion of the teacher's role in implementing the standard. The standards for teacher education complement the performance-based standards of the National Council of Accreditation of Teacher Education (NCATE), which took effect in 2001.

Standards for Teaching Mathematics

Standard 1:	The teacher should pose worthwhile mathematical tasks.
Standard 2:	The teacher's role in discourse should be responsive—posing questions, listening, asking monitoring.
Standard 3	Students' role in discourse should be active and interactive—listening and responding but also questioning, exploring, debating.
Standard 4:	Students should be encouraged to use tools to enhance discourse, including technology, models, writing, visuals, and oral presentations.
Standard 5:	The teacher should create a learning environment that fosters the development of mathematical power.
Standard 6:	The teacher should engage in ongoing analysis of teaching and learning.

Standards for the Professional Development of Teachers of Mathematics

Standard 1:	Mathematics and mathematics education instructors in preservice and continuing education programs should model good mathematics teaching.
Standard 2:	The education of teachers of mathematics should develop their knowledge of the content and discourse of mathematics.
Standard 3:	The preservice and continuing education of teachers of mathematics should provide multiple perspectives on students as learners of mathematics.
Standard 4:	The preservice and continuing education of teachers of mathematics should develop teachers' knowledge of and ability to use and evaluate instructional materials, methods, strategies, and outcomes.
Standard 5:	The pre-service and continuing education of teachers of mathematics should provide them with opportunities to develop and grow as a teacher.
Standard 6:	Teachers of mathematics should take an active role in their own professional development.

Figure 1.12 NCTM Standards for Mathematics Teaching

From National Council of Teachers of Mathematics, Professional Standards for Teaching Mathematics (Reston, VA: Author, 1991), 19-67, 123-73. Reprinted with permission from Professional Standards for Teaching Mathematics, copyright© 1991 by the National Council of Teachers of Mathematics. All rights reserved. Standards are listed with the permission of the National Council of Teachers of Mathematics (NCTM). NCTM does not endorse the content or validity of these alignments.

NCATE's standards call for focusing on student learning; developing meaningful learning experiences; using national and state standards to develop, design, and assess programs; using multiple forms of assessment; emphasizing field and clinical practice; working with diverse student populations; and being committed "to a high quality education for all of America's children" (Mathematical Association of America 2000).

The Conference Board of the Mathematical Sciences (CBMS) seems to be heading in the same direction with their *Mathematical Education of Teachers Project*. The board's (2000) recommendations include:

1. ensuring that future teachers "develop an in-depth understanding of the mathematics they will teach" (1),
2. designing mathematics education courses that "develop careful reasoning and mathematical 'common sense' in analyzing conceptual relationships and in applied problem solving" (2),
3. modeling "flexible interactive teaching," (2)
4. showing "multiple ways to engage students in mathematics" (2).

Teacher Pro-file

Val Penniman
Second and Third Grade

An Elementary Teacher Transformed

Like many elementary school teachers, Val Penniman's own learning experiences in mathematics were somewhat negative:

"I was never a great math student. At one point in algebra class in high school, I was told to stop raising my hand!"

That she can remember this with a laugh illustrates Penniman's current confidence in her accomplishments as a math educator. At this point in her 20-year teaching career in Amherst, Massachusetts, Penniman has been chair of the district-wide mathematics committee. She recently served as the district's first mentor teacher assigned specifically to assist teaches new to the district, and she has developed and marketed an innovative set of calendar-based mathematics materials.

Perhaps even more significantly, Penniman has several years experience as staff member and Director of the Elementary Institute at SummerMath for Teachers (SMT) at Mount Holyoke College. It is this program that Penniman credits with transforming her from an elementary teacher who clung to skill drills and homogenous grouping to one who creates problem-solving lessons for students with a wide range of abilities. Formerly an advocate of text-based teaching, Penniman is now adept at planning classroom activities based on careful attention to what her students tell her about their mathematics understanding.

Making the change was not a quick or easy process

Resisting Change

The first steps of Penniman's journey began in 1989, right as the standards of the National Council of Teachers of Mathematics (NCTM) were being published. At that time, Penniman's building was set up as a multi-age school organized in teaching teams. She and a partner team-taught 50 to 55 children in a combination second and third grade class. Penniman's first contact with SMT was through her teaching partner.

Penniman remembers, "She said that she was going to be taking this course at Mount Holyoke that was going to help her teach math. At that time, we split the students by ability grouping; I took the 'high' kids, and she took the 'low' kids. I can remember the conversation. She was explaining that this was a program that would help her teach students to really understand the concepts about math more than just the skills, and, for instance, they might spend the entire period working on one problem."

She laughs, "We had a big argument. I said, 'No way are they going to learn math if they only do one problem a day. This is crazy!' Infact, it got to the point that she reminded me that I didn't work with the students who 'don't get it' and that I didn't understand. With that challenge, I started working with some of the lower students, too."

By the next summer, Penniman was ready to try the first two-week course at SMT; her teaching partner was scheduled for the second, advanced SMT institute. Penniman was still skeptical, and she later discovered that her partner "actually warned some of the staff that I was coming and that I was going to be a hard sell since I was a complete non-believer at that point."

By the end of her first two-week session at SMT, Penniman relates, "I was pretty much converted." Now she faced the challenge of applying her new beliefs in her classroom.

Making Change Work

Since both Penniman and her partner had been to SMT, they agreed on the changes they wanted to make in their classroom. Says Penniman, "The support for each other was magnificent. We did not break the students into homogeneous groups; we kept them in heterogeneous groups, and that was a very big change."

The two also decided not to use the textbook, which was a tremendous challenge. "It was very hard work," recalls Penniman. "There were no materials out there at that time, so we would write a couple of problems to try out during the day with the students and then we would get together and ask 'What are the next steps? What should we do? Where should we go?' We were just staying about one step ahead."

She describes an example of the type of problem they tried to find: "We have coat hooks out in the hall, and at the beginning of the year we have something like 60 new students in the quad. That year when we were trying to figure out how many coat hooks each kid could have, we looked at each other and said 'We shouldn't figure that out—that is a problem for the students!' Those second and third graders took about a week and a half to solve that one problem."

In addition to the day-to-day challenges of running the classroom, each teacher had internal issues to work through. Penniman recalls, "I had the textbook scope and sequence memorized. I would be comparing, thinking 'If we were using a book, we would be working on this skill. . . .' I was feeling a

Garnering Support for Change

Penniman and her partner found that their second and third graders were quick to accept this way of learning math, and even the parents did not question it. This she attributes to preparation, "At the beginning of the year, we had borrowed a tape from SMT to show to parents at parent night to begin to explain to them what we were doing. We didn't say this is earth shattering. We didn't say this is new math. What we did talk about is children getting to a better conceptual understanding of the math that they were learning."

The school administrators were also receptive. Says Penniman, "The principal didn't really quite understand, but he had heard of SummerMath and he was a supporter. There were a few people who had attended SMT before, so this wasn't a brand new idea. And we didn't talk about it a whole lot to other staff members unless they were interested. It wasn't like we were trying to reform anyone or say this is what you should be doing."

"The principal would sometimes send people in to observe what I was doing. Sometimes they would understand and sometimes they didn't. The assistant principal in our school liked what I was doing a lot, and she would often come in. If there were something fun going on in class, I would invite her in to see it. She encouraged me to do mathematical bulletin boards out in the main hall, which was something we had not done much before."

Committing to Change

It is easy to stay committed to a change decision when things are going well. Then there are the difficult days. Penniman admits, "There were a lot of times when we were stuck. We would say 'What are we doing?' I would be thinking that students were way behind where they should be, or we would be worried about some of the kids who didn't seem to be getting it.

"I can remember one time when I was having a very hard time with getting the students to multiply, and I remember what I call sort of 'falling off the wagon' because I just couldn't get [the concept] across with what we were doing." Penniman showed the children the traditional algorithm, and when she turned around, "these kids are staring at me—just these blank faces, like, 'What are you talking about?'

"I realized then that I couldn't go back to the old method of teaching. We had already made such a change in the way students were learning math that just to get up and show them how to do something wasn't going to work anymore. I can remember thinking, 'Well, we have made the change and I can't just switch back to the way we used to teach.'"

And Penniman would not have it any other way. Her students' scores on standardized tests have remained high and are especially strong in mathematical applications. Even more important to Penniman is the attitude her students have toward mathematics. "Students like getting up and telling the class how they see things. It is empowering when you say to a kid: 'How did you solve this problem?' When the child explains, and the other students listen—hopefully attentively—they are exposed to a new way of solving a problem. Then we all clap."

Source: From Eisenhower National Clearinghouse (ENC) for Mathematics and Science Education, *Teacher Change: Improving Mathematics* (Columbus, OH: Author, 1999).

The task ahead of us, both as a nation and as teachers in the classroom, is neither easy nor simple. The 1996 NAEP found that "46 percent of fourth-grade teachers had little or no knowledge of the standards proposed for mathematics education by the National Council of Teachers of Mathematics; another 32 percent said they were only somewhat knowledgeable" (Hawkins et al. 1998, 41-42). That places 78% outside the mainstream of efforts toward reform. At the same time, polls show that 90% of young people expect their children to attend college and 90% of young people plan to attend college, yet half of these youngsters want to study no more than minimal mathematics and drop the subject altogether as soon as they can. "There is a disconnect about mathematics in this country," said Secretary Riley, (1998) but "Mathematics Equals Opportunity. There could be no more crucial message to send to parents and students of America as we prepare for the coming century."

LOOKING BACK

The twentieth century was a time of continual upheaval in mathematics education. Efforts early in the century to minimalize the curriculum and delay mathematics study gave way at mid-century to experimental curricula and, at the end of the century, to an emphasis on "active mathematical reasoning in elementary school classrooms" (Russell 1999, 1) and changes in educational policies to support curriculum reform in urban and rural areas (Tate and Johnson 1999, 230).

The 20th Century also saw the development of goals and standards that put mathematics education in the public eye. The Nation's Report Card drew attention to achievement levels, and international studies raised concerns and even threatened the public's national pride. The Goals 2000 rallying cry—"be first in the world by 2000"—responded not only to the public's outcry but also to the conviction that a citizenry who understand and can use mathematics is essential to the United States' future. The nation that aspires to lead the world economically, politically, and socially must also lead the world in education, with mathematics and science at the top of the must-know list.

The reform movement gained momentum during the last two decades of the 20th Century. Two outstanding projects among many are AAAS's Project 2061 and NCTM's projects to develop standards for curriculum, teaching, and assessment. Outcomes of these projects have posed both a focus and a challenge for individual teachers and for the profession as a whole.

Questions for Thought, Discussion, and Research

1. Are the mathematics-for-all goals reasonable? Are they achievable? What evidence can you find to support your opinion?
2. Will it be more important to be mathematics literate in the 21st century than it was in the 20th century? Why or why not?
3. What are your own strengths and weaknesses as a learner of mathematics? How might those strengths and weaknesses impact your effectiveness as a teacher? How can you build on the strengths and remedy the weaknesses?
4. When Halley's Comet returns in 2061, will the reformers' vision for mathematics education have been realized? What do you believe, and why do you believe it?
5. How would you define *mathematical literacy*? Do you consider yourself to be mathematically literate? Why or why not?
6. Several of the organizations proposing standards for teacher education suggest the need for teachers to take more mathematics courses and to study the discipline of mathematics as well as methods to teach it. CBMS would like to see elementary school teachers take at least 9 semester hours of mathematics and middle-school teachers, 21 semester hours. Does the proposal have merit? How does it compare to your own program's requirements?